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An overview of the kinematics and workspace of robots with different structures

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Abstract: Robotics is gaining an important place in many areas of industrial production nowadays. With the growing number of robots in production, we are coming to the solution of various tasks to ensure their optimal activity when handling objects or during other technological activities. These are tasks such as planning its movement, planning the robot's trajectory, navigation and tracking the movement of the end member. In this article, we will deal with the issue of robot workspace structures and determining the robot workspace of a specific robot model.

1 Introduction

The development of robots and manipulators belongs to the complex development process of entire mechanical engineering, electrical engineering and many other fields as their inseparable part. Extending the handling capabilities requires the development of new types of kinematic structures, not only of individual types of manipulators, but also of entire handling systems [1-3].

Assessing industrial robots and manipulators requires to distinguish between the mechanical part and the part formed by the control system. A characteristic feature of the mechanical part is the kinematic structure, which significantly affects the basic properties of robots and manipulators, especially the size and shape of the handling and working space.

Currently we have experience with the wide application of industrial automated robotic workstations used mainly for assembly purposes in many manufacturing companies. This results in the need to control an industrial robot in a defined space. The robot mechanism is mostly an open kinematic chain. In the case of a mechanical system of industrial robots used in the production process, this mechanism can have 3 or more degrees of freedom of movement, which depends on the specific robot. In this article, we will focus on the kinematic analysis of manipulator and robot mechanisms. We need to track the position of the working member of the manipulator relative to the inertial coordinate system. Using matrix methods in kinematics, we can determine the position of the end member of the monitored desired working member of the robot. We solve the inverse and forward kinematics. We are also interested in the workspace in which the robot moves during its work. The presented article is dedicated to this issue [4-7].

2 Kinematic structure of robot mechanisms

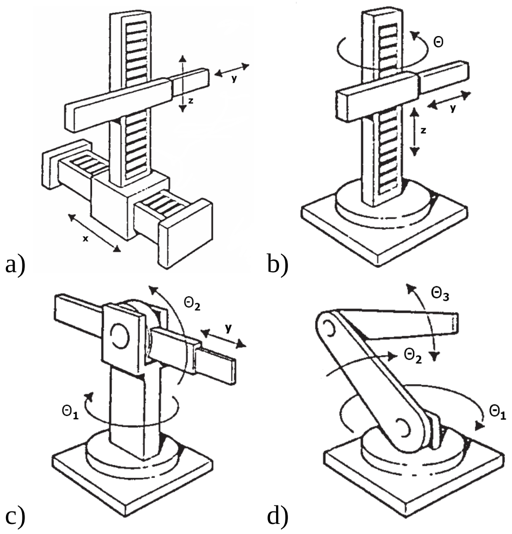
The basic structure of the robot is an open kinematic chain. Alternatively, the chain can also contain a closed loop of members, it will be a kinematic chain with a closed loop. The point of connection of two links is called a kinematic pair. The mobility of the members connected to each other is ensured by joints. Articulation between connected members is realized using a prismatic or rotary joint. The prismatic joint enables the translational movement of the members connected to each other. The rotary joint enables mutual rotation of the members in the joint. Rotary kinematic pairs are preferred over translational ones for compactness and reliability [6].

Our aim is to position and orientation an object in three-dimensional space. We require six degrees of freedom in space, three for the location of a point on the object and three for the orientation of the object relative to the reference coordinate system. If we have more degrees of freedom than necessary, we speak of a kinematically redundant manipulator.

The workspace represents the part of the environment where the end effector of the manipulator can reach. Its shape and volume also depend on the design of the manipulator as well as on the presence of mechanical constraints of the bonds. The task required of the arm is to position the wrist, which is then required to orient the end effector; then at least three degrees of mobility in a three-dimensional workspace are required. The type and sequence of degrees of mobility of the arm, starting from the base joint, allows manipulators to be classified as: cartesian, cylindrical, spherical, SCARA and anthropomorphic [1-7].

3 The configuration of robotic structures

Moving a person from one place to another is a task often solved by a person. In robotics, planning the path along which the working member will move is one of the basic tasks. In robotics, for example, we are looking for the path of a robotic arm or the path of a mobile service robot from the starting position to the target position. During its work, the robot moves in the workspace, which is determined by the possibilities of its joint variables. The types of workspaces are related to the configuration of robotic structures. Their overview is shown in Fig. 1 a) to Fig. 1 d) [7]. The designation of joints in the description of the picture is P - prismatic joint with translational motion, R – rotational joint, with rotational motion [6-7].



a) b) c) d)

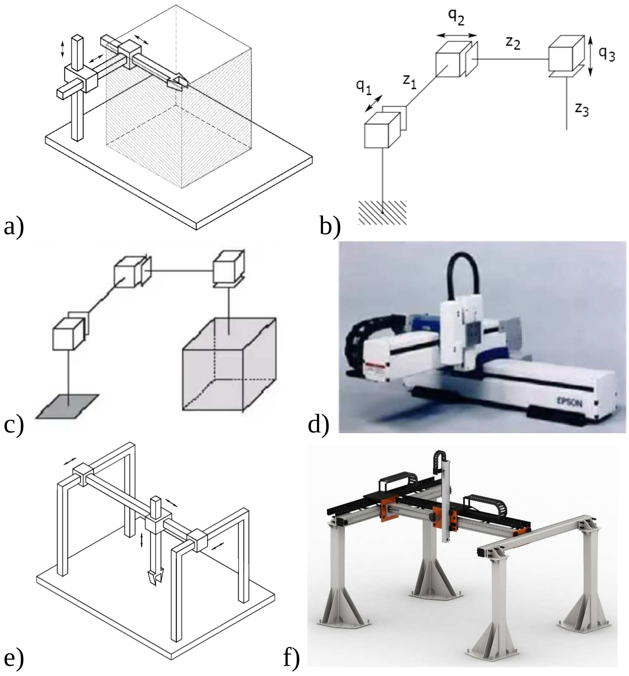
Figure 1 Mechanical structures of robots a) cartesian PPP structure, b) cylindrical RPP structure, c) polar RRP structure, d) universal RRR structure [7]

3.1 *Cartesian geometry of robot*

Cartesian geometry is realized by three prismatic links, the axes of which are typically perpendicular to each other (Fig. 2 a). Due to the simple geometry, each degree of mobility corresponds to a degree of freedom in Cartesian space, so it is natural to make direct movements in space. For a Cartesian manipulator, the joint variables are the Cartesian coordinates of the end effector relative to the base (Fig. 2 b).

The Cartesian structure offers very good mechanical stiffness. The accuracy of wrist positioning is constant in each working space, which is the volume bounded by a rectangular prism (Fig. 2 a). The workspace of the Cartesian manipulator is also shown in Fig. 2 c). An example of a Cartesian robot from Epson-Seiko is shown in Fig. 2 d) [6-7].

In contrast to the high precision, the design has low dexterity because all the joints are prismatic. The approach to handling the object is lateral. On the other hand, if it is desired to approach the object from above, a Cartesian manipulator can be implemented using a gantry structure, as shown in Fig. 2 e). Such a structure makes it possible to obtain a large volume workspace and to handle objects of large dimensions and weight.



a) b) c) d) e) f)

Figure 2 The Cartesian structure of robots with prismatic joints PPP a) kinematic structure and workspace, b) kinematic scheme with generalized coordinates of the kinematic pairs $q_1=d_1$, $q_2=d_2$, $q_3=d_3$, c) workspace of the kinematic structure, d) the Epson Cartesian Robot, e) gantry manipulator [1-7], f) gantry manipulator with 3 axis linear module [14]

Cartesian manipulators are used for material handling and assembly. The joint control of the Cartesian manipulator motors is typically electric and occasionally pneumatic [1-7].

3.2 *Cylindrical geometry of robot*

The cylindrical geometry of the cylindrical manipulator RPP differs from the Cartesian one in that the first prismatic joint is replaced with a revolute joint (Fig. 3 b). If the task is described in cylindrical coordinates, also in this case each degree of mobility corresponds to a degree of freedom. The cylindrical structure offers good mechanical stiffness. Wrist positioning accuracy decreases as the horizontal stroke increases. The workspace is a portion of a hollow cylinder (Fig. 3 a). The horizontal prismatic joint makes the wrist of a cylindrical manipulator suitable to access horizontal cavities. Cylindrical manipulators are mainly employed for carrying objects even of gross dimensions, in such a case the use of hydraulic motors is to be preferred to that of electric motors.

Cylindrical robots operate in a cylindrical-shaped work environment and have at least one rotary axis at the base and one or more prismatic (linear) axes [1-7].

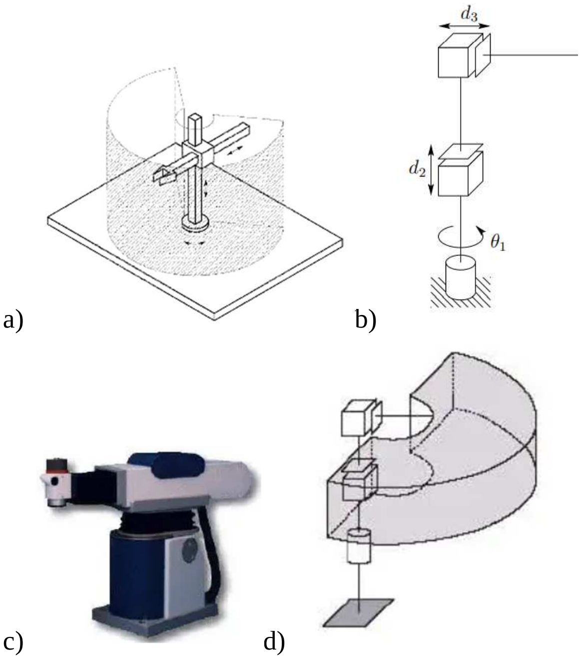


Figure 3 Cylindrical structure of robots a) kinematic structure RPP, b) generalized coordinates of the kinematic pairs are $q_1=\theta_1$, $q_2=d_2$, $q_3=d_3$, c) the Seiko RT 3300 Robot d) workspace of the cylindrical manipulator [1-7]

3.3 *Spherical geometry of robot RRP*

The spherical manipulator RRP differs from the cylindrical one in that the second prismatic joint is replaced with a revolute joint (Fig. 4 a).

Mechanical stiffness is lower than the above two geometries (cartesian and cylindrical) and mechanical construction is more complex. The workspace is a portion of a hollow sphere (Fig. 4 d), it can also include the supporting base of the manipulator and thus it can allow manipulation of objects on the floor. Spherical manipulators are mainly employed for machining. Electric motors are typically used to actuate the joints. Figure 4 e)-f) shows the Stanford arm. They are used in welding, pressure casting and injection or extrusion machines [1-7].

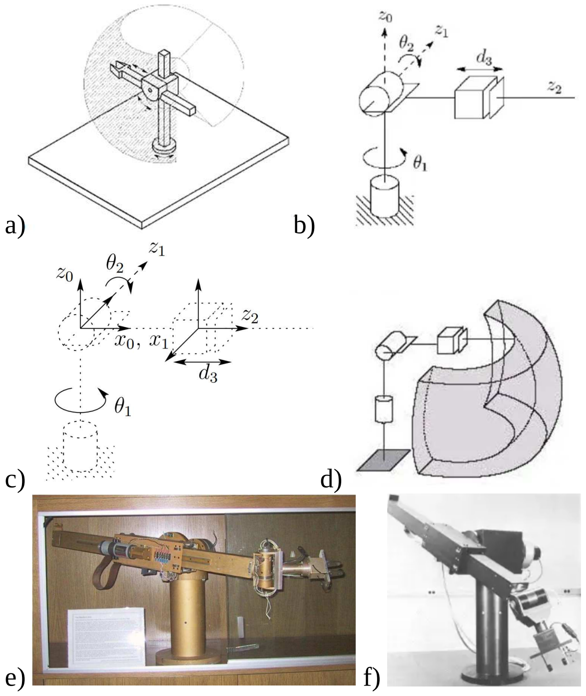


Figure 4 Spherical structure of robots a) kinematic structure RRP, b) generalized coordinates of the kinematic pairs are $q_1=\theta_1$, $q_2=\theta_2$, $q_3=d_3$, c) kinematic scheme of RRP structure, d) workspace of RRP structure, e)-f) Stanford Arm [1-7]

3.4 *Scara manipulator*

A special is the SCARA structure that can be realized by disposing two revolute joints and one prismatic joint in such a way that all the axes of motion are parallel (Fig. 5 b). The acronym SCARA stands for Selective Compliance Assembly Robot Arm and characterizes the mechanical features of a structure offering high stiffness to vertical loads and compliance to horizontal loads. As such, the SCARA structure is congenial to vertical assembly tasks. Wrist positioning accuracy decreases as the distance of the wrist from the first joint axis increases. The typical workspace is illustrated in Fig. 5 a). The SCARA manipulator is suitable for manipulation of small objects; joints are actuated by electric motors [1-7].

SCARA are often used in assembly operations. They vary on the cylindrical design and have two parallel axes that provide movement in one plane.

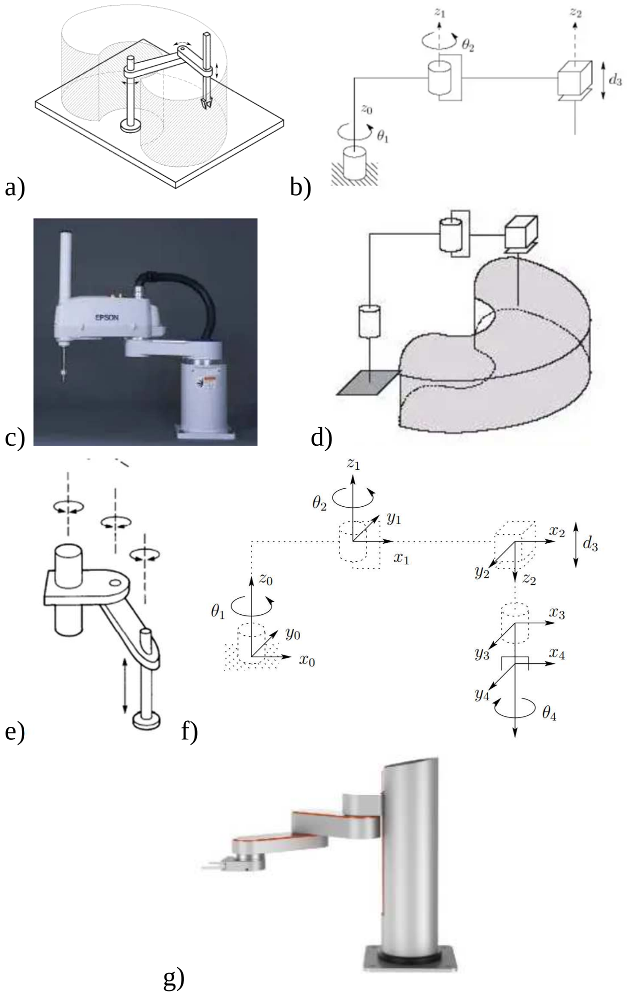


Figure 5 Scara structure of robots a) kinematic structure RRP and workspace, b) generalized coordinates with $q_1 = \theta_1$, $q_2 = \theta_2$, $q_3 = d_3$, c) Epson E2L653S Scara Robot, d) RRP structure and workspace e) kinematic structure RRRP, f) Scara manipulator structure RRPR, g) Scara manipulator structure PRRR [1-7]

3.5 Articulated configuration of robots RRR

The articulated configuration is realized by three revolute joints, the axis of the first joint is orthogonal to the axes of the other two, which are parallel (Fig. 6 b). Based on its similarity to the human shoulder, the second joint is called the shoulder joint and the common joint is called the elbow joint, which connects the "arm" to the "forearm". The articulated structure is the cleverest, as all the joints are rotatable. The accuracy of wrist positioning varies within the workspace. In Fig. 6 a) is the working space part of the sphere [1-7].

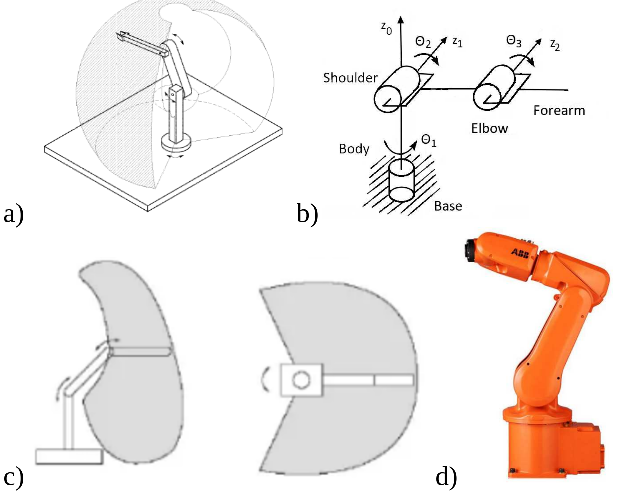


Figure 6 Articulated configuration of robots a) kinematic structure RRR and workspace, b) generalized coordinates $q_1 = \theta_1$, $q_2 = \theta_2$, $q_3 = \theta_3$, c) workspace of the manipulator, d) ABB IRB 120 Robot [1-7]

The joints are usually powered by electric motors. The range of industrial applications of articulated manipulators is wide [1-9].

3.6 Parallel manipulator

Parallel manipulator has two or more independent kinematics chains which are connecting the base to the end-effector (Fig. 7).

A parallel robot with six degrees of freedom, where the mobile platform and the base are connected by six legs, is shown in Fig. 7 d). The desired location of the mobile platform can be achieved by changing the length of the legs using controlled prismatic joints. This architecture was used by Gough in 1947 to design tire testing machines and inspired the design of a flight simulator (Fig. 7 e). It is known as Gough-Stewart parallel robot. Parallel link robots also called Delta robots consist of parallel links connected to a common base. Delta robots are used in pick-and-place systems [1-9].

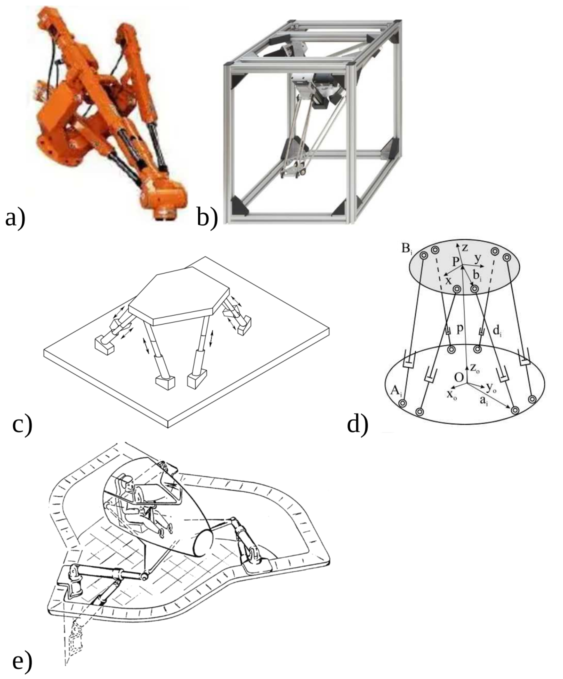


Figure 7 Parallel structure of robots a) Woux - ABB IRB940 Tricept Parallel Robot [6-7], b) Delta robot with number of axes 3, c) scheme of parallel robot [7],d) kinematic scheme of parallel robot [9], e) the design of a flight simulator

4 Kinematic analysis of mechanisms

Today's requirement is to design mechanisms that meet various technological and functional requirements for their operation in the production process. This also applies to the demands for the development of mechanical structures of manipulators and robots. Finding the position of the end member of the kinematic chain of the manipulator of the individual members of the industrial robot based on the known angular rotations in the individual kinematic pairs is a relatively simple task of solving direct kinematics. More demanding is the inverse task of kinematics, where the position and orientation of the gripper are given, and we are looking for the angular coordinates of individual kinematic pairs [4-6]. We encounter the solution of these two tasks most often in the kinematic analysis of robots and manipulators.

In the two-member robotic arm in Fig. 8 with rotational kinematic pairs in individual joints, with arm rotation angles θ_1 and θ_2, we will encounter the solution of the direct and inverse kinematics problem.

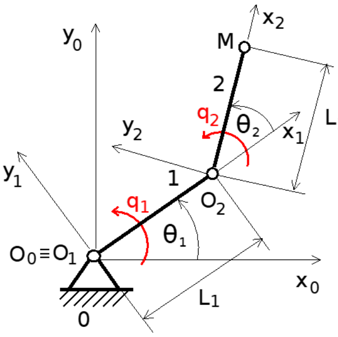


Figure 8 Two-link manipulator kinematic scheme

When solving a direct kinematic problem, the kinematic equations (1) and (2) determine the position of the end point of the arms M.

$$x_M = L_1 cos\theta_1 + L_2 cos(\theta_1 + \theta_2) \qquad (1)$$
$$y_M = L_1 sin\theta_1 + L_2 sin(\theta_1 + \theta_2) \qquad (2)$$

With the known sizes of the angular quantities θ_1 and θ_2, we can determine the position of the point M described by the coordinates x_M and y_M and solve the direct problem of kinematics. With the known position of point M with coordinates x_M and y_M, we can determine the angles θ_1 and θ_2. We solve the inverse problem of kinematics [10-13].

4.1 *The forward kinematics*

The forward kinematics equation establishes a functional relationship between the joint variables and the position and orientation of the end effector. The inverse kinematics problem consists of determining the joint variables corresponding to the given position and orientation of the end effector. The solution of this problem is of fundamental importance for the transformation of the movement specifications assigned to the end effector in the operative space, to the corresponding movements of the joint space, which enable the execution of the desired movement. Regarding the direct kinematic equation (1) and (2) the end-effector position are calculated in a unique way once the joint variables L_1 , L_2 , θ_1 and θ_2 are known.

In the two-member robotic arm in Fig. 8 with rotational kinematic pairs in individual joints, with arm rotation angles θ_1 and θ_2, when solving a direct kinematic problem, kinematic equations (1) and (2) determine the position of the end point of the arms M. This is the solution of a direct kinematic problem.

4.2 *The workspace of robot*

The workspace of an industrial robot can be defined as a set of all end positions of the end of the arm, i.e. flanges that can be reached at the end positions of each of the robot's axes. It is determined by the volume of space within the maximum reach of the robot flange or effector, which is attached to the robot flange (gripper, tool, etc.).

The workspace is an important parameter when planning a robot task. All his movements must remain inside the workspace as shown in Fig. 11 a). It is considered one of the most important characteristics for choosing a suitable robot.

An arbitrary orientation of the end effector is usually not possible in the border positions of the robot workspace. As a rule, it is difficult for the effector to access the manipulated object, which is in location a borderline position of the workspace.

The choice of the workspace depends on the nature of the work tasks, which depend on the used kinematic structure of the robot. Structures with serial kinematics are characterized by the fact that they perform movement independently of each other. The resulting movement is composed of a certain sequence of movements performed by these kinematic pairs.

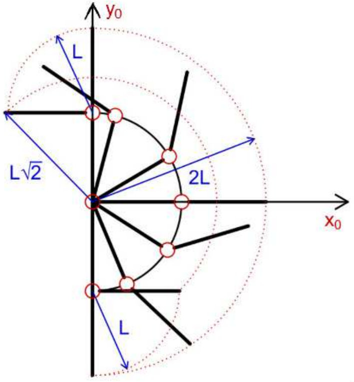
The working space of the manipulator is a subspace of the entire possible space of all arm positions that the end point of the manipulator's gripper can take. The size and shape of the working space of the manipulator depends on the properties of the manipulator, namely on the geometric dimensions of the arms L_1 and L_2 and on the limitations of the joint variables q_1 and q_2. The joint variables in our case are $q_1 = \theta_1$ and $q_2 = \theta_2$.

The following graph (Fig. 11 b) shows all the x,y data points generated by cycling through various combinations of angles θ_1 (theta1) and θ_2 (theta2) and the derivation of the x and y coordinates for the endpoints of the arms. This space can be rendered using the commands shown in Fig. 10 [10-13].

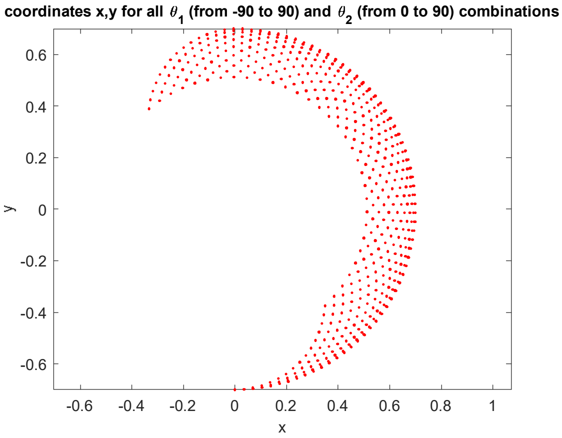
```
figure(2)
L1 = 0.35; % length L1 of first arm
L2 = 0.35; % length L2 of second arm
theta1 = -pi/2:0.1:pi/2; % all possible theta1 values
theta2 = 0:0.1:pi/2; % all possible theta2 values
[THETA1,THETA2] = meshgrid(theta1,theta2); % generate a grid of theta1 and theta2 values
X = L1 * cos(THETA1) + L2 * cos(THETA1 + THETA2); % x coordinates
Y = L1 * sin(THETA1) + L2 * sin(THETA1 + THETA2); % y coordinates
data1 = [X(:) Y(:) THETA1(:)]; % create x y theta1
data2 = [X(:) Y(:) THETA2(:)]; % create x y theta2
plot(X(:),Y(:),'r.','LineWidth',2);
axis equal;
xlabel('x','fontsize',10)
ylabel('y','fontsize',10)
title('coordinates x,y for all \theta_1 (from -90 to 90),\theta_2 (from 0 to
90)','fontsize',10)
text(-0.4,-0.35,'M_0 (x,y) = (0.4,-0.4)'),...
text(-0.4,0.4,'M_{tf} (x,y) = (0,0.6)')
grid on
```

Figure 10 M-file in Matlab m-file in Matlab for determining the workspace of two-link manipulator

The workspace is shown in Fig. 11 b)



a)



b)

Figure 11 Workspace a) two link arms with workspace, b) the workspace of two link arms for the given joint limits of angles θ_{10}=-90°, θ_{20}=0°, θ_{1tf}=90°, θ_{2tf}=90° in Matlab

Workspace with joint restrictions: $0^{\circ} \leq \theta_1 \leq 180^{\circ}$ and $-90^{\circ} \leq \theta_2 \leq 270^{\circ}$is shown in Fig. 12.

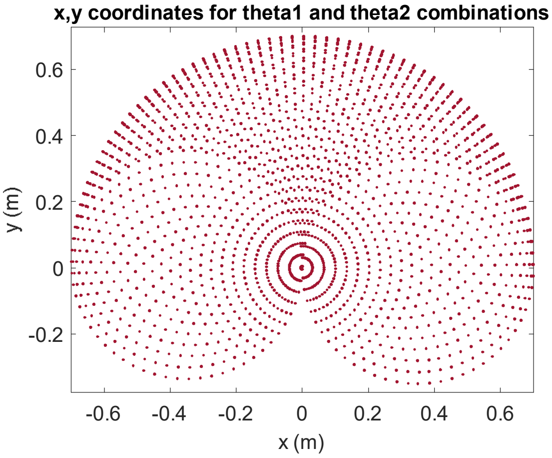
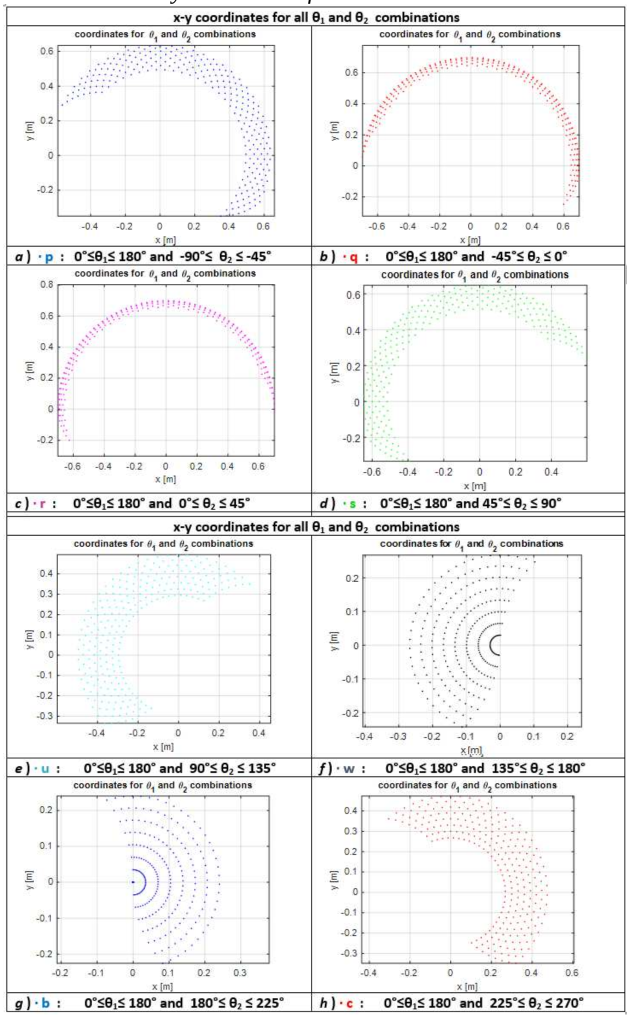


Figure 12 Two-link manipulator workspace with angles in joints $0^{\circ} \leq \theta_1 \leq 180^{\circ}$ and $-90^{\circ} \leq \theta_2 \leq 270^{\circ}$

With the restrictions for θ_1 and θ_2 the workspace will be in the form shown in Tab. 1.

Table 1 The x-y coordinates for all θ_1 and θ_2 combinations



The resulting workspace with joint constraints is shown in Fig. 13.

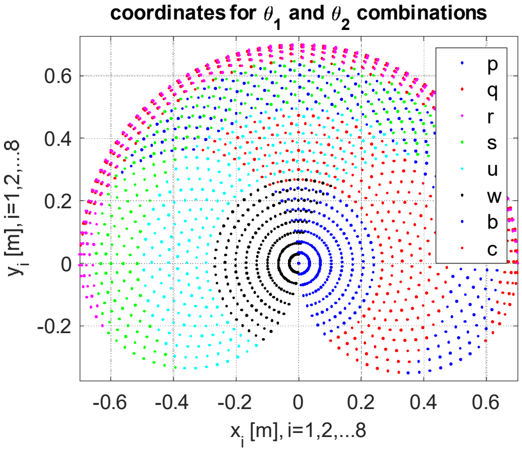


Figure 13 Two-link manipulator – the position of the endpoint

Angles θ_1 and θ_2 of the arms of length L_1 and L_2, at the initial position of the arms and the final position of the arms in Fig. 14 are determined using the m-file in Fig. 16 a).

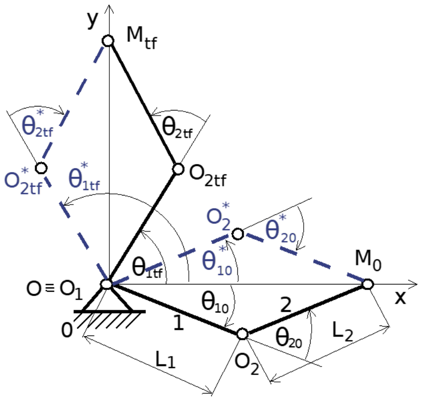


Figure 14 Two link arms in the given joint limits θ_{10}=-90°, θ_{20}=0°, θ_{1tf}=90°, θ_{2tf}=90°

The trajectory of the end point M when moving from position M_0 to position M_{tf} is shown in Fig. 14. The m-file for determining the trajectory when moving from point M_0 to point M_{tf} is in Fig. 16 b) [10-13].

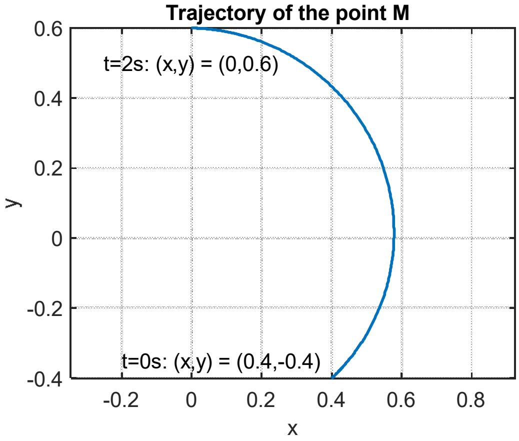
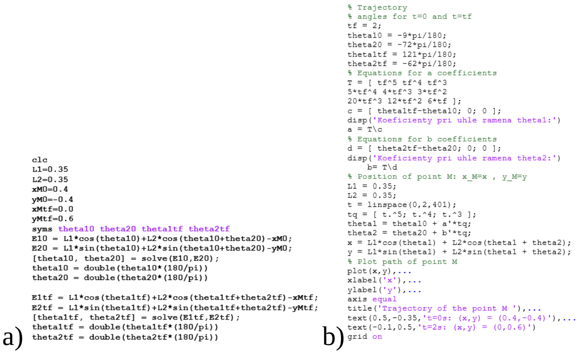


Figure 15 Trajectory of the two-link manipulator from position M_0 (0.4, -0.4) to the position M_{tf} (0, 0.6)



a)

```
clc
L1=0.35
L2=0.35
xM0=0.4
yM0=-0.4
xMtf=0.0
yMtf=0.6
syms theta10 theta20 theta1tf theta2tf
E10 = L1*cos(theta10)+L2*cos(theta10+theta20)-xM0;
E20 = L1*sin(theta10)+L2*sin(theta10+theta20)-yM0;
[theta10, theta20] = solve(E10,E20);
theta10 = double(theta10*(180/pi))
theta20 = double(theta20*(180/pi))

E1tf = L1*cos(theta1tf)+L2*cos(theta1tf+theta2tf)-xMtf;
E2tf = L1*sin(theta1tf)+L2*sin(theta1tf+theta2tf)-yMtf;
[theta1tf, theta2tf] = solve(E1tf,E2tf);
theta1tf = double(theta1tf*(180/pi))
theta2tf = double(theta2tf*(180/pi))
```

b)

```
% Trajectory
% angles for t=0 and t=tf
tf = 2;
theta10 = -9*pi/180;
theta20 = -72*pi/180;
theta1tf = 121*pi/180;
theta2tf = -62*pi/180;
% Equations for a coefficients
T = [ tf^5 tf^4 tf^3
5*tf^4 4*tf^3 3*tf^2
20*tf^3 12*tf^2 6*tf ];
c = [ theta1tf-theta10; 0; 0 ];
disp('Koeficienty pri uhle ramena theta1:')
a = T\c
% Equations for b coefficients
d = [ theta2tf-theta20; 0; 0 ];
disp('Koeficienty pri uhle ramena theta2:')
    b= T\d
% Position of point M: x_M=x , y_M=y
L1 = 0.35;
L2 = 0.35;
t = linspace(0,2,401);
tq = [ t.^5; t.^4; t.^3 ];
theta1 = theta10 + a'*tq;
theta2 = theta20 + b'*tq;
x = L1*cos(theta1) + L2*cos(theta1 + theta2);
y = L1*sin(theta1) + L2*sin(theta1 + theta2);
% Plot path of point M
plot(x,y),...
xlabel('x'),...
ylabel('y'),...
axis equal
title('Trajectory of the point M '),...
text(0.5,-0.35,'t=0s: (x,y) = (0.4,-0.4)'),...
text(-0.1,0.5,'t=2s: (x,y) = (0,0.6)')
grid on
```

Figure 16 a)M-file for θ_1 and θ_2 , b) *M-file for trajectory of two-link manipulator*

The trajectory of the end point M when moving from position M_0 to position M_{tf} (Fig. 15) is in the workspace in Fig. 17 for angular size restrictions:

$$-90^{\circ} \leq \theta_1 \leq 90^{\circ} \text{ a } 0^{\circ} \leq \theta_2 \leq 90^{\circ}.$$

The trajectory in Fig. 17 will be drawn by completing the commands:

```
plot(X(:),Y(:),'r.',x,y,'b-','LineWidth',2),
text(-0.4,-0.35,'M_0 (x,y) = (0.4,-0.4)'),
text(-0.4,0.4,'M_{tf} (x,y) = (0,0.6)')
```

to the above-mentioned m-file for determining the workspace in Fig. 10.

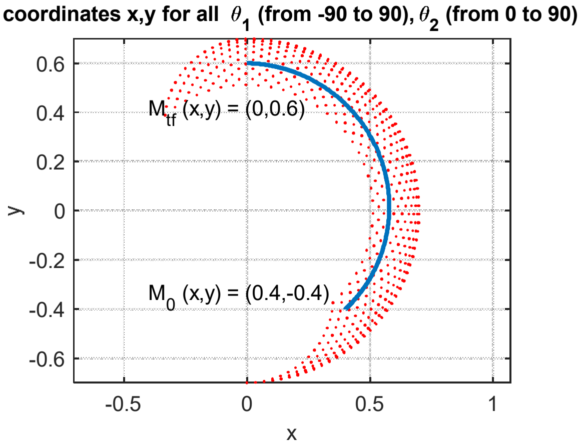


Figure 17 Two link arms and the workspace of the 2R robot for the given joint limits θ_{10}=-90°, θ_{20}=0°, θ_{1tf}=90°, θ_{2tf}=90° in Matlab with trajectory from point M_0 to point M_{tf}.

Various objects, obstacles, tools, etc. can be placed in the robot's handling space. Their position can be easily described with respect to the base space. The forward kinematics is used to determine the relative position of the arm with respect to the objects located in the manipulation space of the robot [6-9].

5 Conclusion

The paper deals with the issue of kinematic analysis of an industrial robot. The kinematic structure of the robot is represented by an open kinematic chain. A direct kinematics problem was used to solve the problem. The solution procedure consists of the calculation and graphical representation of the manipulation space of the end effector while considering various joint constraints. The analysis also includes a graphical representation of the end effector trajectories.

Acknowledgement

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