Acta Mechatronica - International Scientific Journal about Mechatronics
Volume: 6 2021 Issue: 4 Pages: 51-56 ISSN 2453-7306

doi:10.22306/am.v6i4.79

Received: 23 Oct. 2021
Revised: 19 Nov. 2021
Accepted: 10 Dec. 2021

DETERMINING THE STRAIN SENSITIVITY OF RESISTANCE STRAIN GAUGES

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Keywords: strain gauge, k-factor, relative strain

Abstract: When measuring biaxial tension, it is necessary to measure the relative elongation in several directions, for which resistance strain gauges are used. Measurement with resistance strain gauges is based on the change in resistance of the electrical conductor when a deformation occurs. This paper discusses the design of a device for determining the strain sensitivity of resistive strain gauges, which we call the k-factor.

1 Introduction

Measurement with resistance strain gauges is based on the change in resistance of the electrical conductor under deformation. The change in conductivity of metals during deformation was discovered as early as 1856 by Thomson Lord Kelvin. In about 1937, A. Ruge in Massachusetts glued a wire to a paper backing and attached the ends of the wire to feeder wires of larger cross-section. This provided the basis for the manufacture of resistance strain gauges. The rapid development of the aerospace and automotive industries has increased the demands on experimental stress analysis. Resistive strain gauging was one of the avenues. This was driven by the development of new strain gauge apparatus and strain gauges for different purposes. The high measurement accuracy made them attractive in the field of sensor construction in measurement technology.

Measurement with resistance strain gauges is based on the change in resistance of the electrical conductor under deformation. The magnitude of the deformation of a structural element is determined from the change in resistance of the applied strain gauge on the surface under test, which is usually measured when connected to a Wheatstone bridge. Three basic types of strain gauges are used today. These are strain gauges with a wire which is adhered full length to a support, strain gauges in which the winding is formed by etching a pattern from a thin foil, and finally strain gauges in which the wire is fixed by its ends to the support but is free throughout its length [1,2].

For biaxial strain measurements where it is necessary to measure the relative elongation in multiple directions, strain gauges are used that have three or four windings suitably oriented and bonded to a single pad. The windings may be glued not only side by side but also on top of each other. The material of the wire windings is usually chosen from copper-nickel alloy, or nickel-chromium, iron-nickel, or others as appropriate. Each kind has different properties in something. Some have greater sensitivity, others have a greater linear range, others have a higher allowable elongation, they also differ in their physical properties when the temperature changes, etc. [1-4].

1.1 *Physical principles of resistance strain gauges*

The strain gauge strain measurement is based on the assumption that the strain of the object under test is transmitted losslessly to the strain gauge. The prerequisite is a firm connection between the strain gauge and the object to be measured. In most cases, it can only be measured on free and unloaded surfaces of the measured objects. The required firm connection between the object to be measured and the strain gauge is best realized with a

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Volume: 6 2021 Issue: 4 Pages: 51-56 ISSN 2453-7306

special adhesive. In the case of plastic objects or concrete structures, the strain gauge is cast into the model or concrete. In the latter case, special strain sensors in the form of a capsule are required. Other means and methods of fixing are mainly limited to special areas of application, such as ceramic means. For high temperature measurements, spot welding is used. In these cases, special strain gauges are required [3-5].

1.2 Measurement chain

The deformation transmitted from the measured object to the strain gauge causes a measurable change in electrical resistance in an electrical resistance strain gauge. The deformations to be measured by the strain gauge are usually very small, and as a result the changes in resistance are also very small and cannot be adequately measured by the direct use of an ohm meter. It is therefore necessary to use a so-called measuring chain, which allows the small resistance changes of the strain gauge to be determined accurately.

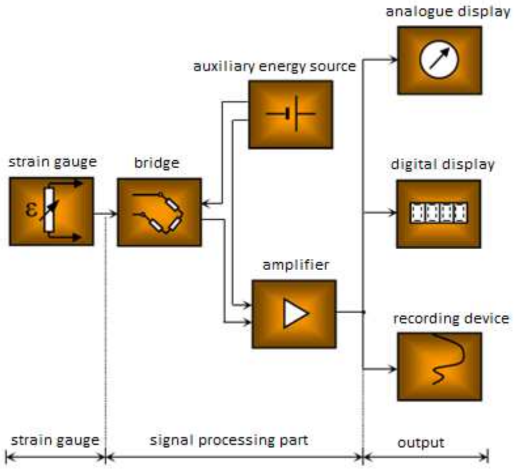

Figure 1 Measurement chain diagram for strain gauge measurement

The above description of the measurement chain shows only schematically the necessary parts. In practice, the measurement chain often contains various accessories such as a device for switching the measuring point, filters, memories for storing maximum values, recorders, etc. In addition, electronic data processing systems may be connected instead of indicating instruments, with various possible uses [2-5].

2 Strain sensitivity of resistance strain gauges

2.1 Longitudinal sensitivity

The operation of resistance strain gauges is based on the Wheatstone and Thomson effect of the interdependence of the relative strain and resistance in an electrical conductor. Each electrical conductor changes its resistance as a result of mechanical loading. The ohmic resistance R (1) of a resistive conductor depends directly on the conductor length l, the specific resistance ρ and indirectly on the cross-sectional area A $\left(R = f(\rho, l, A)\right)$ [1].

Thus

$$R = \rho \frac{l}{A}. \qquad (1)$$

The differential of the above function expresses the change in resistance (2)

$$dR = \frac{\partial R}{\partial \rho} d\rho + \frac{\partial R}{\partial l} dl + \frac{\partial R}{\partial A} dA = \frac{l}{A} d\rho + \frac{\rho}{A} dl - \frac{\rho l}{A^2} dA. \qquad (2)$$

The proportional change in conductor resistance is proportional to the proportional change in conductor length, then (3)

$$\frac{\frac{dR}{R}}{\frac{dl}{l}} = \frac{\frac{d\rho}{\rho} + \frac{dl}{l} - \frac{dA}{A}}{\frac{dl}{l}}, \qquad (3)$$

where $\frac{dl}{l} = \varepsilon$, then strain sensitivity of resistance strain gauges k- factor is (4)

$$k = \frac{\frac{d\rho}{\rho} + \varepsilon - \frac{dA}{A}}{\varepsilon}. \qquad (4)$$

A fully satisfactory explanation for the change in resistivity has not yet been found. De Forest gives a linear dependence between the proportional change in resistance and the proportional change in length by the expression (5)

$$\frac{d\rho}{\rho} = \vartheta \frac{dl}{l} = \vartheta \varepsilon. \qquad (5)$$

Determine the relative change in cross-sectional area of the conductor, Figure 2

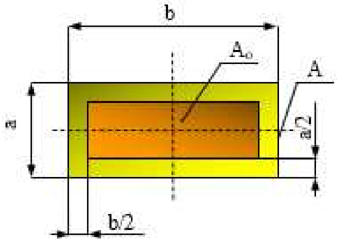

Figure 2 Change in conductor area before and after deformation

The change of area A_0 will be (6)

$$A_0 = (a - \Delta a)(b - \Delta b) = a\left(1 - \frac{\Delta a}{a}\right) b \left(1 - \frac{\Delta b}{b}\right). \qquad (6)$$

We know that $\frac{\Delta a}{a}$ and $\frac{\Delta b}{b}$ is the relative cross-sectional constriction of the conductor, for which we can write (7)

$$\frac{\Delta a}{a} = \frac{\Delta b}{b} = \mu\varepsilon. \tag{7}$$

According to the infinitesimal calculus, we can neglect the very small terms in the above expression ($\mu^2\varepsilon^2 \approx 0$), , the relative change in area will be

$$\frac{dA}{A} = \frac{A_0 - A}{A} = \frac{A(1-2\mu\varepsilon) - A}{A} = -2\mu\varepsilon. \tag{8}$$

Equation (8) is substituted into (4) to obtain

$$k = \vartheta + 1 + 2\mu. \tag{9}$$

In equation (9) for the plastic region, $\vartheta \approx 0$ holds

Bridgman, in his investigation of high pressures in liquids as early as 1917, pointed out the linear dependence of the change in resistance on the all-round pressure, i.e., on the change in the volume of the conductor (10).

$$\frac{d\rho}{\rho} = c\frac{dV}{V}. \tag{10}$$

The volume of the conductor $V = Al$ is a function of the variables $V = f(A, l)$, for which we can construct a differential equation of the form (11)

$$dV = \frac{\partial V}{\partial A}dA + \frac{\partial V}{\partial l}dl = ldA + Adl. \tag{11}$$

Then we get the k-factor in the form (12)

$$k = \frac{\frac{d\rho}{\rho} + \varepsilon - \frac{dA}{A}}{\varepsilon} = c(1-2\mu) + 1 + 2\mu. \tag{12}$$

Since Bridgmann's constant c is constant for a certain material type and processing, the deformation coefficient in this case is just a function of the Poisson number. The expression $c(1-2\mu)$ represents the piezoresistive phenomenon characteristic of semiconductors. The expression $(1+2\mu)$ represents the deformation phenomenon characteristic of electrical conductors.

2.2 Transverse sensitivity

Strain gauges should respond by changing resistance to deformation only in their active direction. The term 'active direction' should be understood for resistance strain gauges to always be in the direction of measurement or the resistance filament of the strain gauge.

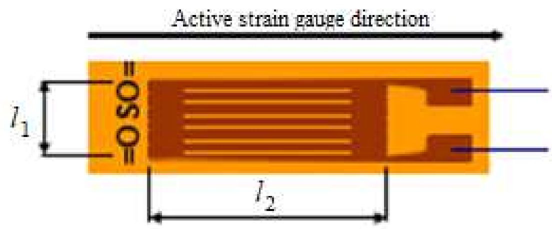

Figure 3 Active strain gauge direction

Consider a winding that is simplified so that the loop is formed orthogonally. The length of the strain gauge is l_1, so the total length of the winding parallel to the longitudinal axis of the strain gauge is nl_1, if n is the number of wires [1].

The length of the strain gauge is determined by the dimension $nl_1 + l_2 = l_{celk}$, where l_1 is the length of the active part of the strain gauge in the longitudinal direction and l_2 is the length of the active part of the strain gauge in the transverse direction. Determine the proportional change in resistance [5,6]

$$\frac{\Delta R}{R} = k\frac{\Delta l_{celk}}{l_{celk}} = \frac{k}{l_{celk}}(\Delta l_1 + \Delta l_2), \tag{13}$$

where Δl_1 is the total elongation of the strain gauge filament, i.e. $\Delta l_1 = n_1 l_1 \varepsilon_1$ and Δl_2 is the total shortening of the filament in the transverse direction, i.e. $\Delta l_2 = l_2\varepsilon_2$. Let us denote by $\varepsilon_1 = \varepsilon$. Then, based on the dependence of the longitudinal elongation of the filament and the transverse shortening of the filament (14)

$$\varepsilon_2 = -\mu\varepsilon_1 = -\mu\varepsilon. \tag{14}$$

The proportional change in resistance can then be expressed as (15)

$$\frac{\Delta R}{R} = \frac{k}{nl_1 + l_2}(n_1 l_1 \varepsilon_1 + l_2\varepsilon_2) = \frac{k}{nl_1 + l_2}(n_1 l_1 \varepsilon - l_2\mu\varepsilon) \tag{15}$$

where

$$K_1 = \frac{kn_1 l_1}{nl_1 + l_2}, \quad K_2 = \frac{kl_2}{nl_1 + l_2}. \tag{16}$$

then

$$\frac{\Delta R}{R} = K_1\varepsilon - K_2\mu\varepsilon = K\varepsilon, \tag{17}$$

$K = K_1(1 - \mu\chi)$ and $\chi = \frac{K_2}{K_1}$ is the transverse sensitivity defined as the ratio of the strain sensitivity K_2 perpendicular to the measurement direction and the strain sensitivity K_1 in the measurement direction.

3 Determination of deflection of a beam with a constant internal bending moment

Determination of the strain coefficient of a resistance strain gauge is most conveniently made by means of a beam subjected to pure bending. The condition for pure bending is that a constant bending moment M_o is applied in the internal cross-section of the beam.

Experimental determination of the k-factor requires determination of the deflection of such a beam.

The progression of the bending moment functions is shown in Figure 4

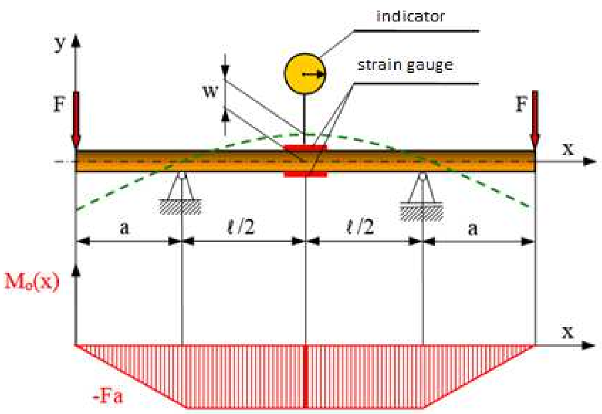

Figure 4 The course of bending moment functions

The deflection of a given beam is expressed by the relation (18)

$$w = -\frac{Fal^2}{8EJ_z}, \qquad J_z = \frac{bh^3}{12}. \tag{18}$$

The deflection *w* itself can be determined by measurement on the beam, for example by using a needle indicator.

The force is (19)

$$F = -\frac{8EJ_z}{al^2}w = \frac{2Ebh^3}{3al^2}w, \tag{19}$$

where w is the value read from the needle indicator.

The next procedure in determining the k-factor requires knowing the relative fiber elongation at the point of strain gauge application. The relative fiber elongation can be determined from Hooke's law. The normal stress from bending in the region under study is (20)

$$\sigma = \frac{M_o}{W_{o_z}} = -\frac{6Fa}{bh^2}, \tag{20}$$

where $W_{o_z} = bh^2/6$ is the cross-sectional modulus of the cross-section of the beam.

The relative deformation after adjustment will be (21)

$$\varepsilon = \frac{-Fa}{W_{o_z}E} = \frac{4h}{l^2}w. \tag{21}$$

The k-factor is usually determined on strain gauges applied to a beam loaded in pure bending. Therefore, it is necessary to wire the strain gauge to sense only small strains due to bending to the exclusion of other types of stresses.

One strain gauge, is glued to the tension side of the beam and the other to the compression side of the beam (Figure 5).

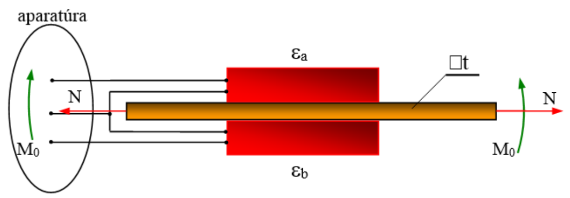

Figure 5 Strain gauge wiring for measuring bending stress excluding tension and temperature change

The strain gauges are connected to a Wheatson bridge, the schematic of which is shown in Figure 6.

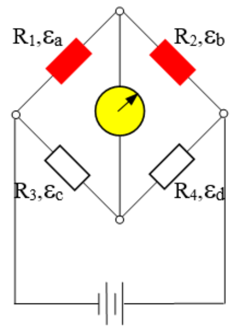

Figure 6 Wheatstone bridge diagram

The bridge consists of four resistive branches, a power supply and a sensitive galvanometer. In two of them we place the strain gauges. In the schematic they are labeled ε_a, ε_b and are represented by resistors R_1 a R_2. The stress increment on the galvanometer can be determined according to the relation (22)

$$dU_g == C(\varepsilon_a - \varepsilon_b - \varepsilon_c + \varepsilon_d), \tag{22}$$

where ε_i, $i = (a, b, c, d)$ are the resistances of the individual branches of the Wheatson bridge.

In the upper branches of the bridge, the resistance changes are due to changes in the length of the outermost fibres on the beam. They can be calculated as follows (23):

$$\begin{aligned} \varepsilon_a &= \varepsilon_t - \varepsilon_o + \varepsilon_T, \\ \varepsilon_b &= \varepsilon_t + \varepsilon_o + \varepsilon_T, \end{aligned} \tag{23}$$

where ε_t is induced by tension, ε_o bending and ε_T by temperature change. The changes in resistance in the lower branches are equal to (24)

$$\varepsilon_c = 0, \ \ \varepsilon_d = 0. \tag{24}$$

ε_c, ε_d are represented by resistors R_3, R_4 and are part of the apparatus. The above circuit is said to be a so-called half-bridge circuit.

By modifying equation (22) we get

$$dU_g = C(-2\varepsilon_o). \tag{25}$$

In the apparatus we use to measure the strain gauge deformation, we set an arbitrary value of the k-factor that is close to 2 and denote it by k_{ap}. We then read off the measured value of the relative strain ε_{ap} from the apparatus. We know that the relative change in resistance

$$\frac{\triangle R}{R} = k_{ap}.\varepsilon_{ap} = k\varepsilon. \tag{26}$$

If the expression (21) is substituted into equation (26) for the proportional deformation, then, after adjustment, we get (27)

$$k = k_{ap}\frac{\varepsilon_{ap} l^2}{4hw}. \tag{27}$$

Rozmery obdĺžnikového priečneho prierezu nosníka sme zvolili $b = 20\ mm$ a $h = 10\ mm$.

The dimensions of the rectangular cross-section of the beam were chosen as b = 20 mm and h = 10 mm.

Figure 7 P-3500 and SB-10 Load Cell and Measuring Assembly

For the experiment, we used the SB-10 (SWITCH A BALANCE UNIT) junction box and the P-3500 (STRAIN INDICATOR) apparatus from Measurements group, Instruments division, Raleigh, NC (Figure 7). The system is designed for static measurements, with the option of connecting strain gauges to a quarter, half, or full bridge [7-11].

A description of the loading device is given in Figure 8. In this device, the load on the beam is induced by a load bolt.

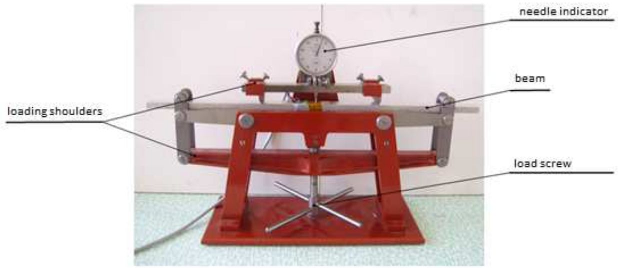

Figure 8 Beam clamped to the loading device

Using the P-3500 apparatus, we read the values of the relative deformation ε_{ap} as a function of the deflection w_i. The measured values are shown in Table 1. The deflection w_i is read using a needle indicator.

Table 1 The measured values

Order of measurement / Measured quantity	1	2	3	4	5	6	7	8	9	10
w_i [mm]	5	10	15	20	25	30	35	40	45	50
$\varepsilon_{ap} \cdot 10^{-6}$	5.3	10.6	16.2	21.5	26.7	31.7	37.8	42.3	48.9	53.5

From the measured values shown in Table 1, we based the calculation of the actual relative deformation ε_{cal} and the corresponding value of the *k*-factor k_{cal}. The value of ε_{cal} (28) was calculated according to equation (21)

$$\varepsilon_{cal_i} = \frac{4h}{l^2} w_i. \tag{28}$$

The *k*-factor values for each measurement will be (29) (see Table 2)

$$k_{cal_i} = \frac{k_{ap} \cdot \varepsilon_{ap_i}}{\varepsilon_{cal_i}}. \tag{29}$$

Table 2 The calculated values

Order of measurement / Calculated quantity	1	2	3	4	5	6	7	8	9	10
ε_{cal_i}	7.9	15.8	23.7	31.6	39.5	47.4	55.3	63.2	71.1	79
k_{cal_i}	1.87	1.88	1.85	1.86	1.87	1.89	1.85	1.89	1.84	1.87

The calculated magnitude of the k-factor was computed using a weighted average and is $\bar{k}_{cal} = 1.88$.

4 Conclusion

The proposed methodological solution procedure for determining the k-factor of strain gauges is correct. The strain device is fully compliant for the determination of the factor. Of course, there are other ways of determining the k-factor , but the above procedure is fast and efficient.

Mastering this methodology provides a suitable tool for solving problems of teaching and practice.

Acknowledgements
This work was supported by the Ministry of Education of Slovakia Foundation under Grant project, VEGA No. 1/0290/18, VEGA No. 1/0389/18 and KEGA No.027TUKE–4/2020, KEGA 030 TUKE-4/2020.

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