doi:10.22306/am.v6i2.76

Received: 19 Apr. 2021
Revised: 05 May 2021
Accepted: 22 May 2021

KINEMATIC ANALYSIS OF THE INDUSTRIAL ROBOT EFFECTOR

Ingrid Delyová
Department of Mechatronics, Faculty of Mechanical Engineering, Technical University of Košice, Park Komenského 8, 042 00 Košice, Slovakia, EU, ingrid.delyova@tuke.sk (corresponding author)
Darina Hroncová
Department of Mechatronics, Faculty of Mechanical Engineering, Technical University of Košice, Park Komenského 8, 042 00 Košice, Slovakia, EU, darina.hroncova@tuke.sk
Peter Frankovský
Department of Mechatronics, Faculty of Mechanical Engineering, Technical University of Košice, Park Komenského 8, 042 00 Košice, Slovakia, EU, peter.frankovsky@tuke.sk

Keywords: kinematic analysis, transformation matrix, joint, arm of robot, effector

Abstract: The technical level of industrial robots and manipulators is rapidly increasing, thus supporting the expansion of their application space. The requirements of the industry are various special manipulations with objects, guiding the end effector of the robot along the prescribed trajectory at a given speed while maintaining the angular position and orientation of the object. The paper presents a survey of a robot with a kinematic scheme formed by an open kinematic chain with revolute joints.

1 Introduction

The development of robots and manipulators belongs to the complex development process of entire mechanical engineering, electrical engineering and many other fields as their inseparable parts. Extending the handling capabilities requires the development of new types of kinematic structures, not only of individual types of manipulators but also of entire handling systems [1-3].

Assessing industrial robots and manipulators requires distinguishing between the mechanical part and the part formed by the control system. A characteristic feature of the mechanical part is the kinematic structure, which significantly affects the basic properties of robots and manipulators, especially the size and shape of the handling and working space.

1.1 Kinematic investigation of mechanisms

In the kinematic investigation of mechanisms, we encounter two types of tasks:

- Kinematic analysis of mechanisms
- Kinematic synthesis of mechanisms

Kinematic analysis of mechanisms of given dimensions investigates the relationship between position, speed and acceleration of driven and driving members. If we investigate only the trajectories of individual members of the mechanism and their points corresponding to the given movement of the driving members, we speak of forwarding kinematics of trajectories [1,3].

Kinematic synthesis of mechanisms is to design mechanisms that meet various technological and functional requirements for their operation in machines, devices and their parts. Finding the position of individual members of an industrial robot at known coordinates of individual kinematic pairs is a relatively simple task. A more difficult task is the inverse position problem, where the position and orientation of the gripper are given, and we look for the coordinates of individual kinematic pairs [4-6].

1.2 The forward kinematics

Various objects, obstacles, tools, etc., can be placed in the robot's handling space. Their position can be easily described with respect to the base space. The forward kinematics is used to determine the relative position of the arm with respect to the objects located in the manipulation space of the robot for the given articulated coordinates. The relationship between generalized articulated coordinates and coordinates of the base space is nonlinear (1), (2):

$$q_j = q_j(r_i), \tag{1}$$

or

$$q_j = q_j(x_i, y_i, z_i), \tag{2}$$

where q_j are generalized coordinates, $j = 1,2,, \dots, n$,
j is the number of degrees of freedom,
i is the number of members $i = 1,2, \dots, m$.

A change in the joint coordinates means that, in general, neither the trajectory of the effector point is straight nor the orientation of the effector is linear. This nonlinearity is based on the fact that the functions between the articulated and external (global) coordinates consist of expressions containing trigonometric functions. The solution is in the creation of a kinematic model of the robot. There are several methods for building such a model. The most common method is the Denavit-Hartenberg kinematic modelling convention. It connects the local coordinate system with each member. The method of connection depends on the homogeneous transformation matrix [1-2].

1.2.1 Denavit-Hartenbergová convention

Let’s consider an open kinematic chain with *n* members and a coordinate system to which we assign the respective kinematic parameters [5-7]. Each member of the chain is characterized by two dimensions, a common normal distance b_j along a common normal between the axes of the joints *i* and *i*-1 and the other dimension is the angle of rotation β_j between these axes in a plane perpendicular to b_j.

We assign a coordinate system O_i,x_i,y_i,z_i to each member and a generalized coordinate q_i to each joint, which is defined in the axis of rotation. The axis z_i is oriented in the direction of the *(i+1)-th* axial joint. The axis x_i is normal to the z_{i-1} and z_i and is oriented from joint *i* to joint *i*+1. The axis y_i complements the rectangular, right-handed coordinate system.

1.2.2 Homogeneous transformation matrix

Homogeneous transformation matrices contain information about the rotation between two coordinate systems and information about the distance between their origins. The purpose of introducing these matrices is to allow a more compact notation of position vectors expressed in different coordinate systems. The position vector $^{i-1}\boldsymbol{r}_A$of point A with respect to the system *i*-1 can be expressed by the relation (3)

$$^{i-1}r_A = {}^{i-1}A_i \cdot {}^{i}r_{i-1,i}, \tag{3}$$

where

$^{i}\boldsymbol{r}_A$ is the position vector of point A with respect to the system *i*,

$^{i}\boldsymbol{r}_{i-1,i}$ is the vector of the distance between the origin of the system *i* with respect to the system *i*-1,

$^{i-1}\boldsymbol{A}_A$ is the rotation matrix between the system *i* and *i*-1.

The homogeneous transformation matrix is expressed by the relation (4), (5)

$$^{i-1}T_i = \begin{pmatrix} ^{i-1}A_i & ^{i}r_{i-1,i} \\ 0 \quad 0 \quad 0 & 1 \end{pmatrix}. \tag{4}$$

We then write the position vector

$$^{i-1}r_A = {}^{i-1}T_i \cdot {}^{i}r_A. \tag{5}$$

2 Kinematic structure of the robot

The kinematic structure of the robot is represented by an open kinematic chain with six degrees of freedom of movement [8-9]. The chain consists of two basic parts. The first part comprises members 1, 2, 3, has three degrees of freedom of movement and serves to position the manipulated object, fig. 1. The second part serves for the orientation of the gripper and thus the manipulated object in space, it comprises members 4, 5, 6 and also has three degrees of freedom of movement. Member 1 is the stand rotatably mounted to the base plate, Fig. 1. An arm is connected to the stand with a shoulder joint 2. There is another arm connected to this arm with elbow joint 3. Each joint has a limited range of rotation. The investigated mechanism consists of rotating kinematic pairs only. One of the members of the robot is a gripper, which represents a multi-member mechanism. It has an associated coordinate system that defines its orientation in space.

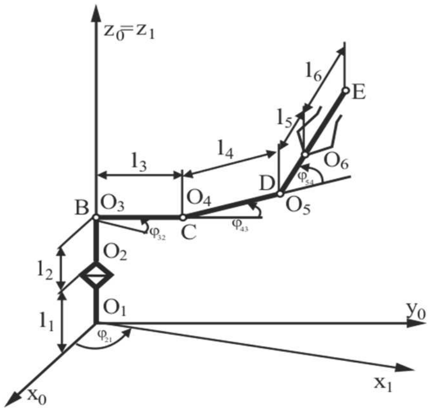

Figure 1 Robot diagram (B - shoulder joint, C - elbow joint, D - wrist joint, E - effector endpoint

Homogeneous transformation matrices have a relatively complex expression, so we introduce an abbreviated notation of a function (6), (7).

$$c_i = \cos q_i, \qquad s_i = \sin q_i, \tag{6}$$

$$c_{ijk} = \cos(q_i + q_j + q_k), \; s_{ijk} = \sin(q_i + q_j + q_k), \tag{7}$$

The transformation matrix T_{17} of the whole robot's kinematic chain is obtained as a product of the individual transformation matrices (8), (9).

$$T_{17} = T_{12} \cdot T_{23} \cdot T_{34} \cdot T_{45} \cdot T_{56} \cdot T_{67} \tag{8}$$

$$T_{17} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{9}$$

We used following parameters for members of the robot:

Table 1 The four parameters of robot of DH convention

i	β_i [°]	b_i $[mm]$	θ_i [°]	d_i $[mm]$
1	$\frac{\pi}{2}$	0	q_1	l_1
2	0	l_2	q_2	0
3	0	l_3	q_3	0
4	$\frac{-\pi}{2}$	0	q_4	l_4
5	$\frac{\pi}{2}$	0	q_5	l_5
6	0	0	q_6	l_6

The position of point E with respect to the reference coordinate system is given by the equation (10), (11).

$$ {}^{1}r_E = T_{17} \cdot {}^{7}r_E \tag{10} $$

$$ \begin{pmatrix} {}^{1}\mathbf{r}_{Ex} \\ {}^{1}\mathbf{r}_{Ey} \\ {}^{1}\mathbf{r}_{Ez} \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} {}^{7}\mathbf{r}_{Ex} \\ {}^{7}\mathbf{r}_{Ey} \\ {}^{7}\mathbf{r}_{Ez} \\ 1 \end{pmatrix} \tag{11} $$

Simultaneous start-up in individual kinematic pairs with constant angular acceleration $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$

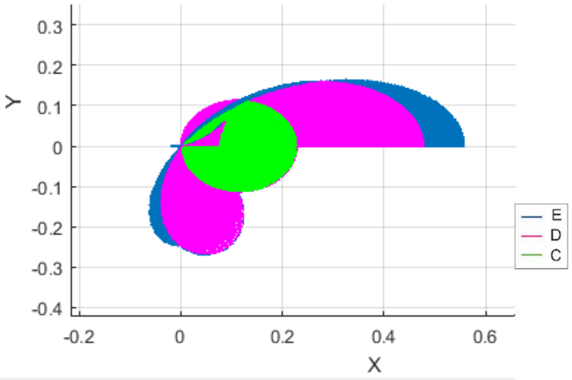

Figure 2 The trajectories of individual members

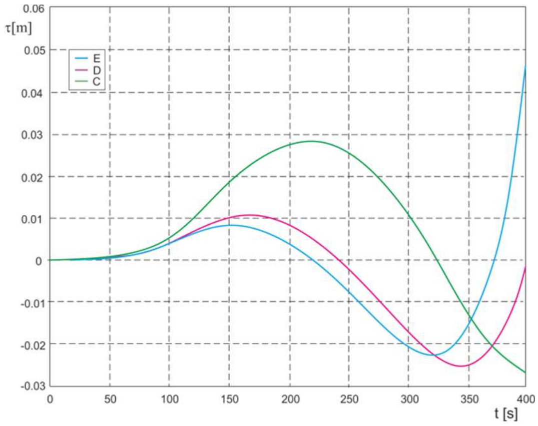

Figure 3 Radius of torsion of joint trajectory

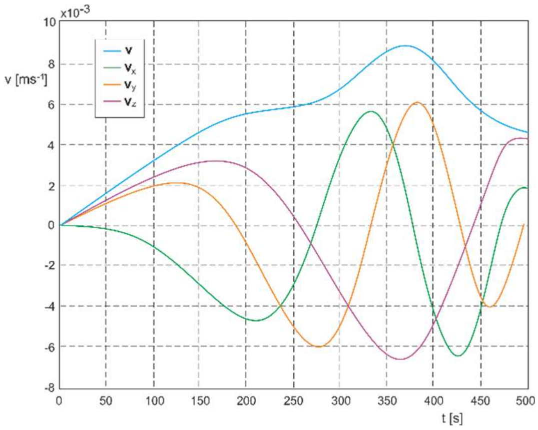

Figure 4 Joint E velocity at constant angular accelerations

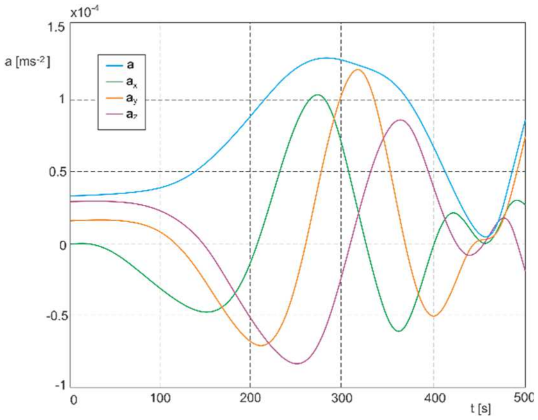

Figure 5 Joint E acceleration at constant angular accelerations

Figure 6 shows the acceleration of joint E at constant angular accelerations for which it applies $\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4$. Figure 7 shows the acceleration of the joint E at angular accelerations $\alpha_1 = 0$, $\alpha_2 > \alpha_3 > \alpha_4$.

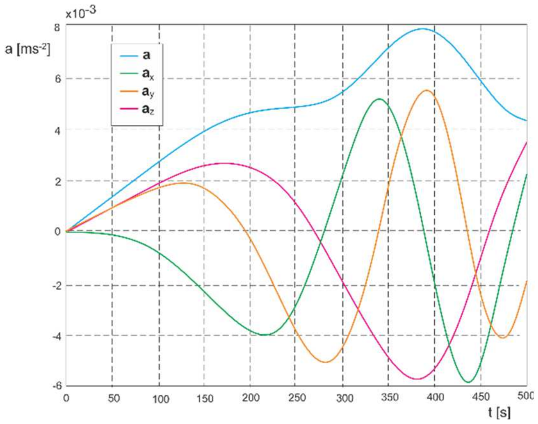

Figure 6 Joint E acceleration

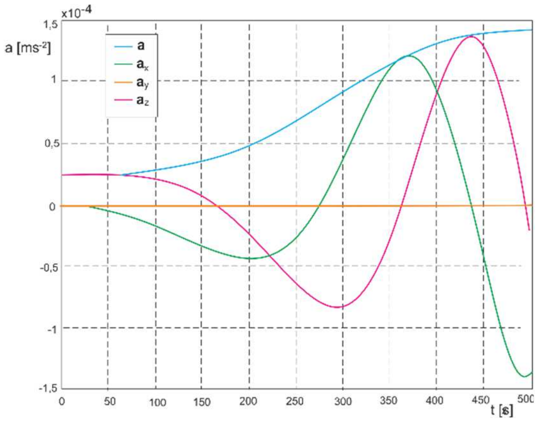

Figure 7 Joint E acceleration

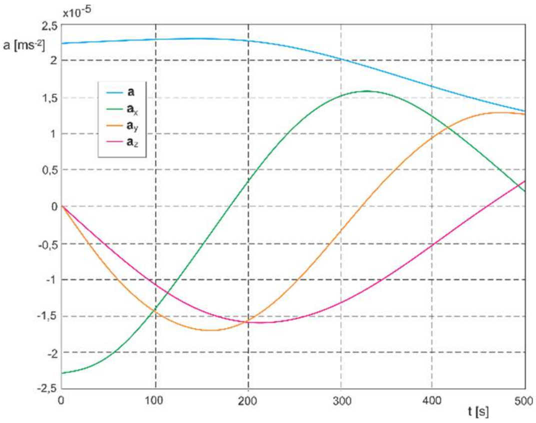

Figure 8 Joint E acceleration at constant angular velocity

Figure 8 and Figure 9 show the acceleration of the joint E with simultaneous starts of individual kinematic pairs with constant angular velocities. The acceleration for $\omega_1 = \omega_2 = \omega_3 = \omega_4$ is in Fig. 8, for $\omega_1 > \omega_2 > \omega_3 > \omega_4$ is in Fig. 9

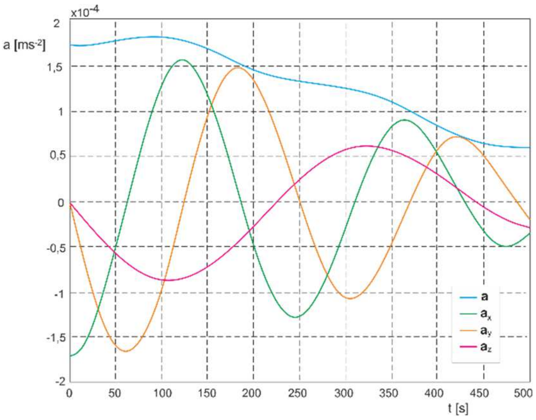

Figure 9 Joint E acceleration at $\omega_1 > \omega_2 > \omega_3 > \omega_4$

3 Conclusion

The paper deals with the issue of kinematic analysis of an industrial robot. The kinematic structure of the robot is represented by an open kinematic chain. The matrix method was used to solve the problem. The solution procedure consists of construction of the transformation matrices of coordinate systems and graphical representation of the manipulation space of the end-effector. The analysis also includes graphical representations of some kinematic quantities and also the evaluation of the influence of the parameters of the mechanism, which influence their values.

Acknowledgements

This work was supported by the Ministry of Education of Slovakia Foundation under Grant project, VEGA No. 1/0290/18, VEGA No. 1/0389/18 and KEGA No.027TUKE–4/2020, KEGA 030 TUKE-4/2020.

References

[1] TALÁCKO, J., STEJSKAL, S., STEJSKAL, V.: *Průmyslové manipulátory a roboty I.*, ČSVTS, Praha, 1986. (Original in Czech)

[2] VUKOBRATOVIČ, M.: *Intoduction to robotics*, 1989.

[3] HRONCOVÁ, D., RÁKAY, R., LIPTÁK, T.: Sim Mechanics and Forward and Inverse Problem of Dynamics, *Journal of Automation and Control*, Vol. 3, No. 3, p. 58-61, 2015.

[4] HRONCOVÁ, D., FRANKOVSKÝ, P., BETTES, G.: Kinematical Analysis of Crank Slider Mechanism with Graphical Method and by Computer Simulation, *American Journal of Mechanical Engineering*, Vol. 4, No. 7, p. 329-343, 2016.

[5] HRONCOVÁ, D., DELYOVÁ, I., FRANKOVSKÝ, P.: Kinematic Analysis of Mechanism Using MSC Adams, *Applied Mechanics and Materials*, Vol. 611, No. 7, pp. 83-89, 2014.

[6] DELYOVÁ, I., HRONCOVÁ, D., FRANKOVSKÝ, P.: Analysis of Simple Mechanism Using MSC Adams, *Manufacturing Technology*, Vol. 14, No. 2, pp. 141-145, 2014.

[7] KELEMENOVÁ, T., FRANKOVSKÝ, P., VIRGALA, I., MIKOVÁ, Ľ, KELEMEN, M.: Machines for inspection of pipes, *Acta Mechatronica*, Vol. 1, No. 1, pages 1-7, 2016.

[8] PIRNÍK, R., HRUBOŠ, M., NEMEC, D., BOŽEK, P.: Navigation of the autonomous ground vehicle utilizing low-cost inertial navigation, *Acta Mechatronica*, Vol. 1, No. 1, pp. 19-23, 2016.

[9] LIPTÁK, T., KELEMEN, M., GMITERKO, A., VIRGALA, I., HRONCOVÁ, D.: The control of holonomic system, *Acta Mechatronica*, Vol. 1, No. 1, pp. 15-20, 2016.

[10] CRANE III, C.D., DUFFY, J.: *Kinematic analysis of robot manipulators*, Cambridge University Press, 2008.

[11] DASH, K.K., CHAUDHURY, B.B., SENAPITI, S. K.: *Inverse Kinematics Analysis of an Industrial Robot Using Soft Computing*, In: International Conference on Application of Robotics in Industry using Advanced Mechanisms, pp. 270-279, Springer, 2019.

[12] SINGH, T.P., SURESH, P., CHANDAN, S.: Forward and inverse kinematic analysis of robotic manipulators, *Inernational Research Journal of Engineering and Technolnogy*, Vol. 4, No. 2, pp. 1459-1468, 2017.

Review process

Single-blind peer review process.