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DETERMINATION OF POISSON NUMBER AT THIN ROD-SAMPLES WITH NON-STANDARD CROSS-SECTIONS BY PENDULUM METHODS

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Abstract: The paper describes the measurements of modulus of elasticity of thin samples and related Poisson number by one device – Searle´s pendulum. We have focused our attention mainly to non-traditional samples with non-standard (i.e. other than circular) cross-sections.

1 Introduction, what is Poisson number?

The Poisson number μ belongs to basic physical constants characterising the elastic properties of solids. It is defined as the ratio of the relative transverse shortening and the relative longitudinal prolongation. This can be expressed by means of elastic modules as

$$\mu = \frac{E}{2G} - 1 , \quad (1)$$

where *E* means the tensile modulus (or Young´s modulus) and *G* is the shear modulus.

In our task we have used a device that is able to measure both of these relevant quantities. This device is so-called "Searle´s pendulum", designed by American physicist G.F.C. Searle [1]. This device is commonly used to measure Young's modulus of thin specimens with classical circular cross-sections. We have extended this use to the measurement of samples with other cross-sections (some of them with a hollow character), and we used the vertical arrangement of the system to measure the shear modulus of elasticity *G*, too. It also presents a convenient way to determine the Poisson number μ.

2 Theoretical analysis and experimental procedures

Searle´s pendulum - in its classic form - is based on two flywheels connected by the sample being measured to create an oscillating system after deflection. The device can be used in two configurations: horizontal and vertical (Fig. 1a) and 1b)).

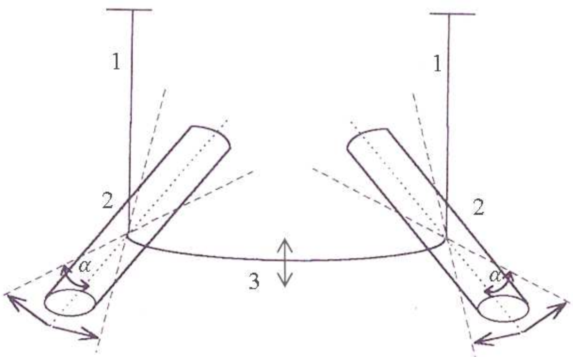

Figure 1 a) Horizontal flywheel set.
1 – hanging threads, 2 – cylinder flywheels, 3– measured wire (arrows indicate the direction of the oscillations, α is an angle of deflection)

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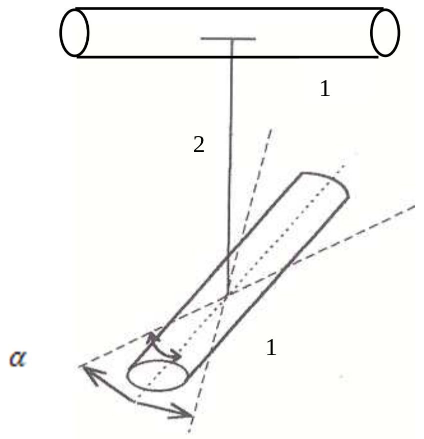

Figure 1 b) Vertictal flywheel set.
1 – cylinder flywheels, 2 – measured wire (arrows indicate the direction of the oscillations)

The principle of operation is essentially the same in both configurations: the vibration energy of the sample is "spilled" into the vibration of the flywheels, and vice versa. The sample performs bending oscillations in the first case and torsional ones in the second case. This can be illustrated by following scheme:

horizontal configuration → bending vibrations → E

and

vertical configuration → torsion vibrations → G

Dynamic analysis of pendulum operation results in relationships for flexibility modules [2]

$$E = \frac{4\pi^2}{T^2} \cdot \frac{lJ}{2J_A} \tag{2a}$$

and

$$G = \frac{4\pi^2}{T_G^2} \cdot \frac{lJ}{2J_A} \tag{2b}$$

Here J is the momentum of inertia, l presents the length of sample and T_E and T_G are the periods of oscillations, respectively. J_A represents the area moment of inertia (see below).

But now here we must differentiate the type of flywheels, too. In the case of cylindrical flywheels, as well as in our picture, the moment of inertia is given by the known relationship

$$J = m\left(\frac{L^2}{12} + \frac{R^2}{4}\right). \tag{3a}$$

The parameters R and L are the diameter and length of flywheels, and m means their (single) mass. The next possible cases would be:

Square flywheels

$$J = \frac{1}{12} m(A^2 + B^2), \tag{3b}$$

(A and B are the length and the width of the prism), and dumbbell flywheels

$$J = \frac{1}{4} mL^2, \tag{3c}$$

(L means the length of dumbbell).

The quantity of J_A represents the area moment of inertia with respect to the bending axis. This variable is different for otherwise shaped cross-sections, and we will pay more attention to it in the next part of the article; it will be summarized in Table 2 in detail.

3 Experimental part

3.1 The experimental assembly for measurements

We used the apparatus, that photo is – in horizontal arrangement – illustrated in Figure 2.

It consists from two homogeneous steel rollers in the role of flywheels, each having a mass m = 0.72 kg, a length L = 137 mm and a radius R = 14,6 mm. Size of moment of inertia of each of them, determined from the relation (3a), had a value of $J = 1.15 \times 10^{-3}$ kg.m^2.

We had used both - horizontal and vertical - configuration of device.

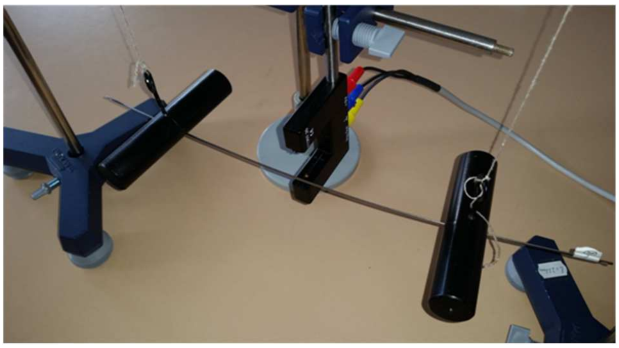

Figure 2 Experimental assembly in horizontal arrangement. Vibrating wire sample crosses the infrared beam of an optical

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sensor (prismatic body with the shape of figure U in the centre of operation)

The arrangement of this assembly when measuring the torsion module is analogous, but it is oriented in the vertical direction.

We have performed measurements of several types of samples. First we measured samples with a classic full circular cross-section, then we measured samples with unconventional cross-sectional character.

3.2 Samples with classic (circular) cross-sections

We performed test measurements for standard circular cross-sectional samples. We used thin wires of different materials and cross-sections.

The moment of inertia of such samples is

$$J_A = \frac{1}{4}\pi r^4 \tag{4}$$

After its incorporation into relations (2a) and (2b) we get expressions for the modules of elasticity

$$E = \frac{8\pi l}{r^4 T_E^2} \tag{5}$$

and

$$G = \frac{8\pi l J}{r^4 T_G^2} \tag{6}$$

The pendulum times T_E and T_G, that are important in these measurements were scanned electronically.

All the samples had the same "active" length (i.e. the distance between the points of attachment to flywheels) l = 0,30 m, and the same radius r = 1 mm. A review of results of measurements of them is given in Table 1.

Table 1 The results of measurements of samples with standard circular cross-sections, diameter r = 1 mm

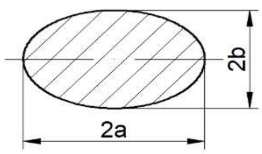
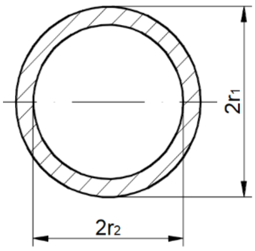
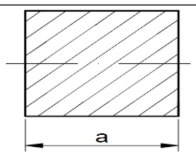
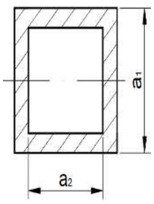
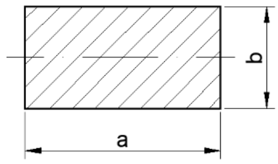
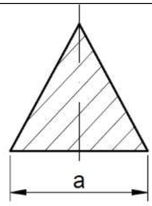
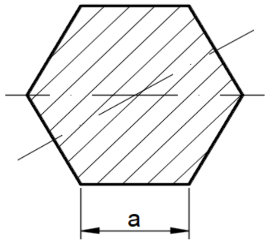
Sample	Period of bending oscillations T_E (s)	Period of torsion oscillations T_G (s)	Tensile modulus *E* (GPa)	Shear modulus *G* (GPa)	Poisson number	
					Measured μ	Table valued parameter
Steel	0.238	0.374	202	81	0.28	0.28-0.31
Copper	0.303	0.499	123	45	0.34	0.34-0.35
Alluminium	0.399	0.654	71	26.5	0.32	0.27-0.30
Brass	0.338	0.557	99	36.5	0.36	0.35-0.37
Polyamid (nylon 6,6)	2.32	3.88	2.1	0.75	0.40	0.39-0.41
PVC	1.97	3.41	2.9	1.01	0.42	0.40-0.42
Polystyrene	1.73	3.94	3.7	1.37	0.35	0.34-0.37
Polypropylene	2.51	4.31	1.8	0.66	0.37	0.36-0.41
Polyethylene	2.66	4.53	1.6	0.56	0.41	0.42-0.46

3.3 Samples with unconventional cross-sections

Using this method, we had measured the tensile modulus of several unconventional metal and plastic samples, and one wooden specimen, moreover.

An overview of these patterns being possible is summarized in Table 2.

Table 2 Overview for different possible cross-sectional shapes of wire samples

	Cross-section	Area moment of inertia
Ellipse		$J_A = \frac{1}{4}\pi a^3 b$
Circle - hollow		$J_A = \frac{1}{4}\pi(r_1^4 - r_2^4)$
Square - full		$J_A = \frac{1}{12}a^4$
Square - hollow		$J_A = \frac{1}{12}(a_1^4 - a_2^4)$
Rectangle		$J_A = \frac{1}{12}ab^3$
Triangle (equilateral)		$J_A = \frac{\sqrt{3}}{96}a^3$
Hexagon (regular) side and/or top axis		$J_A = \frac{5\sqrt{3}}{16}a^4$

In the last column there are presented the relations for the calculation of area momentum of inertia, that stands out in expressions for receiving the modules *E* and *G*.

An overview of the measured samples, including the relevant geometric parameters and the values being obtained, is given in Table 3.

Table 3 Parameters and results of measurements of samples with non-standard cross-sections

Sample	T_E (s)	T_G (s)	*E* (GPa)	*G* (GPa)	μ	μ_{tab}
Steel – circle hollow r_1 =1,1 mm; r_2 = 0,75 mm	0.195	0.312	204	79.8	0.28	0.28-0.30
Polypropylene – circel hollow r_1 = 1,5 mm; r_2 = 1 mm	1.12	2.00	1.7	0.61	0.39	0.36-0.41
Polystyrene – ellipse *a* = 1,5 mm; *b* = 1 mm	1.58	2.61	2.3	0.84	0.37	0.36-0.40
Polyethylene – ellippse *a* = 1,5 mm; *b* = 1 mm	1.89	3.22	1.6	0.55	0.45	0.42-0.46
Polystyrene – square full *a* = 2 mm	1.49	2.43	2.4	0.9	0.33	0.36-0.40

Polyamide – square full *a* = 1,5 mm	2.77	5.28	2.1	0.71	0.41	0.39-0.41
Polypropylene – square hollow a_1 = 2 mm; a_2 = 1 mm	1.74	3.39	1.8	0.62	0.45	0.36-0.41
Polypropylene – rectangular *a* = 2,7 mm; *b* = 1 mm	1.56	2.60	1.7	0.63	0.38	0.36-0.41
PVC – rectangular *a* = 2,5 mm; *b* = 1,2 mm	1.25	2.12	2.8	0.97	0.44	0.38-0.43
Polyamide (nylon) – triangle *a* = 2,5 mm	2.14	3.59	2.1	0.75	0.40	0.38-0.42
Polyamide – hexagonal *a* = 1,5 mm	2.97	5.54	2.0	0.73	0.37	0.39-0.41

3.4 Simplification of the evaluation process

We have determined the Poisson number through the elastic modules *E* and *G*.

However, there exists also an easier way: After substituting expressions for *E* and *G* from (6) and (7) to (1), the expression will be considerably simplified to form (6) and (7) to (1), the expression will be considerably simplified to form

$$\mu = \frac{T_G^2}{T_E^2} - 1 \quad . \qquad (7)$$

In this way, we should eliminate “uncomfortable” quantities *E*, *G* and J_A from the calculations.

4 Conclusion

Searle's pendulum, though a simple and fairly accurate device to measure elastic modulus, is used relatively few (unfairly in our opinion). Additionally, it should be mostly in the cases of "slow oscillating" samples of circular cross section.

The equipment being described is simple and illustrative, completing the range of pendulum-based methods for the measurements of elasticity constants. It does non-require intricate measuring equipment and works without destruction, practically. Even extremely thin samples can be measured without a risk of damage or permanent deformation. The activity of pendulums is stable, the system phases do not “tune out” or dump even after several tens or hundreds of oscillations.

The results are sufficiently precise, as evidenced by the fact that the measured values are well correlated with the table values. The measurement error did not exceed 8% accuracy to statistical calculations [3].

This method can be used successfully in the wires, plastic and textile industries (investigation of elasticity of thin materials [4]), in botany (elasticity of stalks) and the like. As so as a demonstration chapter in university textbook (section of “Vibrating Movements” or “Solid State Physics”), or a task for laboratory exercises [5].

Acknowledgment

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