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STIMULATED DECELERATION OF BENDING VIBRATIONS AS A METHOD FOR DETERMINING THE TENSILE MODULUS OF THIN ROUND CROSS-SECTIONAL SAMPLES

Karol Kvetan
Slovak University of Technology, Faculty of Materials Science and Technology, Institute of Materials Science, Bottova 25, 917 24 Trnava, Slovak Republic, EU, karol.kvetan@stuba.sk (corresponding author)
Marián Kubliha
Slovak University of Technology, Faculty of Materials Science and Technology, Institute of Materials Science, Bottova 25, 917 24 Trnava, Slovak Republic, Automation and Mechatronics, Bottova 25, 917 24 Trnava, Slovak Republic, EU, marian.kubliha@stuba.sk
Janette Kotianová
Slovak University of Technology, Faculty of Materials Science and Technology, Institute of Applied Informatics, Bottova 25, 917 24 Trnava, Slovak Republic, EU, janette.kotianova@stuba.sk
Ondrej Bošák
Slovak University of Technology, Faculty of Materials Science and Technology, Institute of Materials Science, Bottova 25, 917 24 Trnava, Slovak Republic, EU, ondrej.bosak@stuba.sk

Keywords: tensile modulus, dynamic methods, thin samples, one-, three- and four-point bending vibrations, stimulated deceleration, flywheels, additional slowing bodies
Abstract: The slowing of bending vibrations in the determination of the tensile modulus of thin round samples is at the centre of interest .We describe the utilization of this method at the measurements by means of classical one-and three-point bending, and we introduce a new approach – a four-point bending.

1 Introduction

Tensile modulus (also called elastic modulus or Young's modulus) *E* belongs to the most important mechanical material constants. It characterizes their stiffness and flexibility properties.

For the long thin samples such as rods, wires, strings, fibers, stalks, and the like, dynamic methods are often used. The best known ones are one-point and three-point bending vibration methods. Here are advantageous the facto, that relevant quantities, i. e. periods and oscillation amplitudes are relatively easy to measure, and therefore no complicated apparatus is needed. Moreover - for a comparison with static methods - no force load is required, so there is no risk of irreversible deformations [1], [2].

However, in the case of fast-oscillating specimens, difficulties arise. Vibrations are too fast and therefore difficult to measure. This problem is manifested at coarser and shorter samples, primarily.

For these reasons, it is preferable to introduce some **stimulated deceleration of oscillating processes.**

The principle of our method is that we provide a slowing of vibrations by means of additional bodies or flywheels. They take part of the energy and slow down the oscillatory process so that vibrations can be monitored by the naked eye. In addition - the system will vibrate more stable and longer.

The main idea is the same as when measuring the elastic modulus in the shear using torsion pendulums; here the oscillation velocity is buffered by the additional bodies, too [1].

2 Theoretical analysis and experimental procedures

We have applied this principle for three types of measurements: first for classical one-point and three-point bends, and then we have extended this idea to an unconventional four-point bend (for terminological information - the so-called two-point bending does not exist).

2.1 One-point bending

This can be done by a constraining the specimen of length *l* into a solid wall in the perpendicular direction (Fig. 1).

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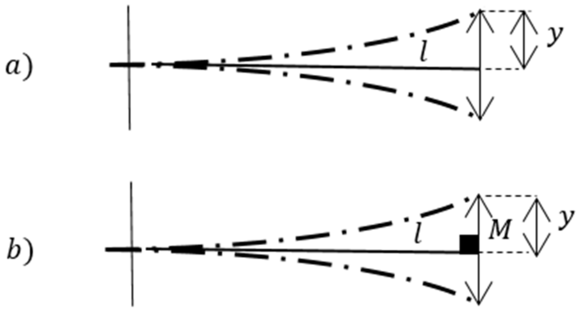

Fig. 1. Vibrations at one-point bending: a measured rod sample a) without additional body b) with additional slowing weight M

When the free end is bent by force F and let go, the sample will perform free oscillations with period T_0. When an additional body with a mass of M is attached to the free endpoint, the system will oscillate more slowly with the period T_1. After measuring these variables, we can determine the tensile modulus E based on the following derivation:

As it results from the theory of elasticity, a relation between the applied force F and the bending of the sample y is

$$F = \frac{3EJ_A}{l^3} y \qquad (1)$$

The quantity of J_A means the areal moment of inertia with respect to the bending axis. At the same time, the equation of motion applies

$$F = m_r a \qquad (2)$$

where m_r means so-called a reduced mass of the rod (note that this quantity is not identical to its actual mass, and this quantity is unknown to us).

The acceleration a in vibrational motions is

$$a = \omega^2 y = \frac{4\pi^2}{T_0^2} y \qquad (3)$$

By combining these relationships we get for E

$$E = \frac{4}{3} \frac{\pi^2 l^3}{J_A T_0^2} m_r \qquad (4)$$

The areal moment of inertia for samples of a circular cross section with a radius r is

$$J_A = \frac{1}{4} \pi r^4 \qquad (5)$$

Using this relationship we shall get the final relationship after this assignment

$$E = \frac{16}{3} \frac{\pi l^3}{r^4 T_0^2} m_r \qquad (6)$$

The problem is that there acts a quantity of m_r, that can not be determined by weighing. Its calculation would be difficult, and moreover it is not a constant, but this variable varies with other parameters.

Therefore it is preferable to remove this variable from the relevant calculations. This can be achieved by a simple way:

We add a (point) weight with the mass M at the end of the rod and we re-vibrate the stick. Now the stick will vibrate slowly with the period T_1. Currently, we can re-write the relationship (6) into the shape

$$E = \frac{16}{3} \frac{\pi l^3}{r^4 T_1^2} (m_r + M) \qquad (7)$$

By dividing equations (6) and (7) we get

$$\frac{T_1^2}{T_0^2} = \frac{m_r}{m_r + M} \qquad (8)$$

From here we can express a reduced mass

$$m_r = \frac{T_0^2}{T_1^2 - T_0^2} M \qquad (9)$$

We put this expression into relationship (6) for calculating the modulus of elasticity. By this way we get

$$E = \frac{16}{3} \frac{\pi l^3}{r^4 (T_1^2 - T_0^2)} M \qquad (10)$$

Using this method, we had measured the tensile modulus of several metal and plastic samples, and one wooden specimen, moreover. They all had the same lengths $l = 40$ cm, but different radii. We used the wearer's body with the mass $M = 3$ g as the weights. The relevant results are summarized in Table 1.

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Table 1 Parameters of samples and the results of measurements at one-point bending

Sample	Period T_0(s)	Period T_1(s)	Modu -lus E (GPa)	Table valued parame- ter E_{tab} (GPa)
Steel I r = 1,00 mm	0.160	0.203	210	180 – 210
Steel II r = 1,25 mm	0.107	0.135	196	180 – 210
Copper r = 1 mm	0.223	0.281	110	103 – 126
Alluminium r = 2 mm	0.114	0.126	68	65 – 70
Brass r = 1,5 mm	0.072	0.131	101	86 – 105
PVC r = 1,1 mm	1.133	1.429	2.9	2,5 – 3,0
Polyamide r = 1,0 mm	1.586	2.031	2.1	1 – 2,6
Polystyrene r = 1,5 mm	0.539	0.680	3.7	3.2 – 3.5
Oak wood – r = 2 mm (along the fibres)	0.214	0.262	8.8	10 – 13

2.2 Three-point bending

The oscillation deceleration is created by two flywheels that are suspended on rigid threads and attached to the endpoints of the sample (Fig. 2). These bodies take a part of the energy of motion and vibrate concurrently with the oscillations of the samples.

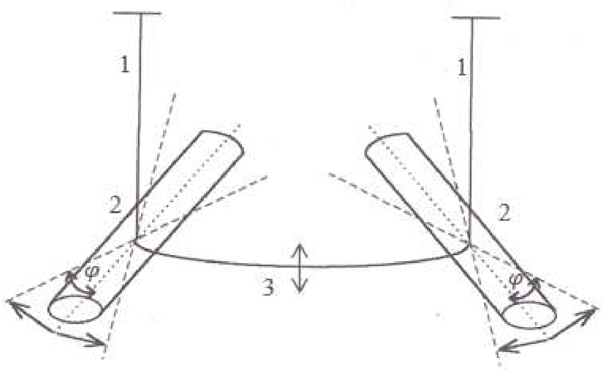

Fig. 2. Three-point bending pendulum with the slowing flywheels
1 – hanging threads, 2 – cylinder flywheels, 3 – measured sample (arrows indicate the direction of the oscillations)

This process is described by the equations valid for oscillation processes

$$\frac{d^2\varphi}{dt^2} + \frac{2EJ_A}{lJ}\varphi = 0 \tag{11}$$

$$M = -J \cdot \varepsilon \tag{12}$$

and

$$M = K \cdot \varphi = \frac{2EJ_A}{l}\varphi \tag{13}$$

Here φ means an angle of deflectin, $\varepsilon = d^2\varphi/dt^2$ is the angular acceleration of vibrations, l is the length of sample, M is the mechanical moment of the rotation force and K is so-called directional moment, i.e. moment of power, required for bending of wire by unit angle.

There are two important momentum variables in the relationships:

- the quantity of J_A means the areal moment of inertia; this quantity is already described in detail in section 2.1
- the moment of inertia J of the damping flywheels with respect to the perpendicular center axis. In the case of cylindrical flywheels, as well as in our picture, it is given by known relationship

$$J = m\left(\frac{L^2}{12} + \frac{R^2}{4}\right) \tag{14}$$

The parameters R and L are the diameter and length of flywheels, and m means their (single) mass.

These equations result for the circular frequency ω

$$\omega = \sqrt{\frac{K}{J}} = \sqrt{\frac{2EJ_A}{lJ}} \tag{15}$$

After using the known relation between circular frequency and the oscillation period T

$$\omega = \frac{2\pi}{T} \tag{16}$$

we shall get

$$E = \frac{4\pi^2}{T^2} \cdot \frac{lJ}{2J_A} \tag{17}$$

and - after fitting from (4) - we shall get the resulting relationship

$$E = \frac{8\pi l J}{r^4 T^2} \tag{18}$$

The desired modulus of elasticity thus can be determined by measuring the oscillations of the sample in particular.

We performed measurements of several samples of wires with different radii of cross sections. All the samples had the same "active" length (i.e. the distance between the points of attachment to flywheels) l = 0.4 m. Elastic

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modulus values were determined from the relationship (18), where we set the magnitude of the momentum of inertia a value of J = 1.15x10^{-3} kg.m^2 (as it follows from (14)).

An overview of the measured samples, including the relevant geometric parameters and the measured values, is given in Table 2.

For information, we provide that this method can be used successfully in the wires, plastic and textile industries (investigation of elasticity of thin materials [3]), in botany (elasticity of stalks) and the like.

Table 2 Parameters of samples and the results of measurements at three-point bending

Sample	Period of oscillations T (s)	Modulus of elasticity (measured) E_{meas} (GPa)	Table valued parameter E_{tab} (GPa)
Steel I r = 1,00 mm	0.239	203	180 – 210
Steel II r = 1,25 mm	0.153	201	180 – 210
Copper r = 1 mm	0.313	118	103 – 126
Alluminium r = 2 mm	0.150	64	65 – 70
Brass r = 1,5 mm	0.148	104	86 – 105
PVC r = 1,1 mm	1.679	2.8	2.5 – 3.0
Polyamide r = 1,0 mm	2.292	2.2	1 – 2.6
Polystyrene r = 1,5 mm	0.807	3.5	3.2 – 3.5
Oak wood – r = 2 mm (along the fibres)	0.392	9.4	10 – 13

2.3 Four-point bending

We have suggested new equipment for examination of flexure properties of full circular samples by slowed oscillations – a vertically coupled gravitation set (also known as *coupled reverse pendulums*). Unlike the previous device, the slowing flywheels are located in the vertical direction; the purpose is also to use gravitational forces. A direction the measured sample can be arbitrary – horizontal or vertical; we used a horizontal way.

However, the theoretical analysis is more complicated – we need to consider – except for gravity – also the contribution of next factors (see Section 3.3).

The sample has a full circular shape with the diameter R. Vibration deformations are caused by the action of a pair of side forces F (Fig.3).

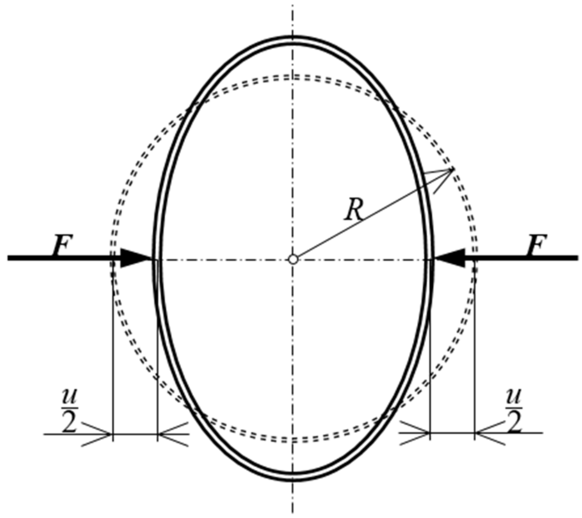

Fig. 3. Vibration deformation at four-poind bend by acting the force F. Using the deformation forces F the radius size changes by u/2

The compressive effect of the forces is achieved by two rods of the gravitational torsional pendulum. By this way the sample plays the role of coupling here.

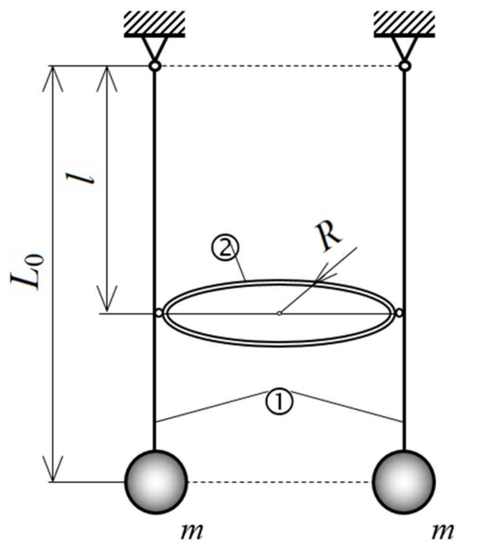

Figure 4 Measuring equipment scheme.
(1 - pendulums, 2 – circular hoop connecting pendulums)

Determining the value of the module is challenging here; we present a deduction in Article [4]. The resulting relationship has a shape

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$$E = \frac{8(\pi^2 - 8)mgL_0R^3}{\pi^2 l^2 d^4}\left[\frac{T_1^2}{T_2^2} - 1\right] \qquad (19)$$

m is the mass of the pendulum, g is the gravitational acceleration, L_0 is distance of the pendulum centre from the rotation axis, l is distance of the wire connection from the pendulum point and d is a wire diameter. Symbols T_1 and T_2 have the significance of the pendulum vibration period itself, and a pair of pendulums braked by their mutual coupling.

We have investigated the elastic properties of several materials – metallic and plastic - all with the same geometric parameters (length l and diameter d). These parameters are summarized in Tab.3.

Tab.3 Geometrical parameters used in our experiments

L_0 [m]	m [kg]	d [mm]	R [m]	l [m]
0.84	0.87	1.5	0.16	0.25

Also the period of oscillation of the pendulum itself was the same $T_1 = 1.040$ s. So, the only varying parameter had been the period of braked vibrations T_2. The corresponding results for these quantities are summarized in Tab. 4, together with the relevant values of tensile modules E calculated from the formula (19).

Tab.4 Quantities measured for the determination of elastic modulus by the four-point bending

Sample	T_2 [s]	E [GPa]	E_{tab} [GPa]
Steel	0.610	203.3	200 - 210
Aluminium	0.808	69.9	67 - 70
Copper	0.707	124.1	110 -120
Brass	0.759	101.7	90 - 100
Polyamide (nylon)	1.034	1.3	0.9 – 1.4
Polystyrene	1.031	1.8	1.5 - 1.8

3. Conclusion

We have presented the results of the measurements at single-, three- and four-point bends, taking into account the specifics of the individual processes.

The results are sufficiently precise, as evidenced by the fact that the measured values are well correlated with the table values [5], and with each other, too.

Statistical calculations - according to the theory of measurements - showed an uncertainty of about 8 % for our experiments. These values are comparable - or just a little worse - than values obtained by static methods. But, on the other hand, they have two significant benefits:

1. The speeds of vibrations are diminished by means of flywheels, which is particularly valuable for samples with fast free oscillations. The time periods are therefore easier to measure.

2. Our systems are phase-stable, they do not “tune-out” even after several tens or hundreds of oscillations.

The benefit is also the possibility of simple measuring the samples with non-standard cross-sections (hollow, square, elliptic, heqahonal, triangular etc.) without the risk of permanent damage.

These methods can be used as demonstration tasks in physics exercises, (see, for example [6]), or as part of the teaching chapters “Physics of Solids” or “Vibration Movements”.

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