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SELECTION OF THE CRACK DRIVING FORCE CONCEPT IN CONTEXT OF LINEAR-ELASTIC AND ELASTIC-PLASTIC FRACTURE MECHANICS

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Abstract: Fracture mechanics is a continuum mechanics approach to describe cracks in materials. There are plenty of fracture mechanics concepts such as linear elastic fracture mechanics (LEFM), elastic-plastic fracture mechanics (E-PFM), dynamic, the time-dependent fracture mechanics that are limited to specific loading conditions, crack geometry (length) and material behaviour. Current paper evaluates applicability of a crack driving force in context of LEFM and E-PFM for arbitrary (quasi-static) loading and yielding conditions to help engineers choose appropriate fracture mechanics concept for their applications.

1. Introduction

Two fracture mechanics approaches are standardly used in LEFM – The energy release rate and the stress intensity factor. Both hold their validity for linear-elastic material behaviour or small scale yielding (SSY) conditions. The machine parts are components are normally designed for linear-elastic behaviour. Hence, the stress intensity factor is widely used concept in the mechanical engineering. Due to the overloading and local stress/strain field around notches are machine parts exposed to local plastic deformations. The E-PFM concepts are incorporated to describe behaviour of such cracks. The crack tip opening displacement (CTOD) and the J-integral require additionally lower restriction on the fracture mechanics tests as the stress intensity factor approach [1]. The cyclic CTOD can be used for characterization of a crack growth under large scale and general yielding condition under cyclic loading [2] but that approach is limited to fracture Mode I; for Mode II (or Mixed Mode) are currently investigated approaches like crack face displacement [3]. The J-integral does not characterize the real crack driving force for a crack in an elastic-plastic material behaviour even for proportional cyclic loading [2], [4]. The physically appropriate crack driving force for an elastic-plastic material behaviour has been derived [4]. Based on the configurational force concept ("modified J-integral"), the integration contour which encloses the active plastic zone characterizes the physically appropriate crack driving force [4] but it can lacks in practical applications, where the active plastic zones originates from the crack cannot be separated from other source of plastic deformation in the system [5].

The fracture mechanics describes as continuum mechanics tool behaviour of physically long cracks. The physically (or micromechanically) small cracks are often studied experimentally [6], [7], [8]. Here has to be said, that a threshold value of a small crack is an open issue [9] new method to asses a crack initiation and crack growth of a small crack are in the development [10], [11], [12].

According to a literature survey, The configurational force concept is able to describe physically appropriate crack driving force for arbitrary (quasi-static) loading and yielding conditions but has drawback in some practical applications [5].

The most work was done as part of the PhD. thesis [13]. Unless stated other, the figures are taken from [13] without reference.

2. Crack driving force

According to [1] the crack with initial length a_0 in a loaded body will be extended if the "generalized crack driving force" D_{gen} is equal or larger than a "generalized crack growth resistance"

$$D_{gen} \geq R_{gen}. \tag{1}$$

The crack driving force is generally a loading parameter for the crack that tries to elongate the crack. The crack driving force comes from the work of the applied forces and (or) from the stored strain energy in the body. The crack growth resistance is a function of the material and impedes the crack extension. The fracture toughness is a material property and is measured by a fracture mechanics test where a sample with a sharp (pre-fatigued) crack with the initial length a_0 is loaded. In the test are measured the load point displacement v, the load F and the crack growth extension Δa. The crack driving force increases with loading and if $D_{gen} = R_{gen}$ crack extension occurs. The crack growth toughness $R_{gen}(\Delta a)$ is obtained in a similar way from an equilibrium $D_{gen} = R_{gen}$.

The crack growth resistance is a constant for linear elastic material behaviour. Then LEFM can be used. LEFM can be applied also in the case when the plastic zone r_{pl} is

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considerably smaller than the crack length a and the sample length b. That condition is called small-scale yielding (ssy) [1], see Figure 1:

$$a, b \gg r_{pl}. \tag{2}$$

If large-scale (lsy) or general yielding conditions prevail (gy) E-PFM must be used.

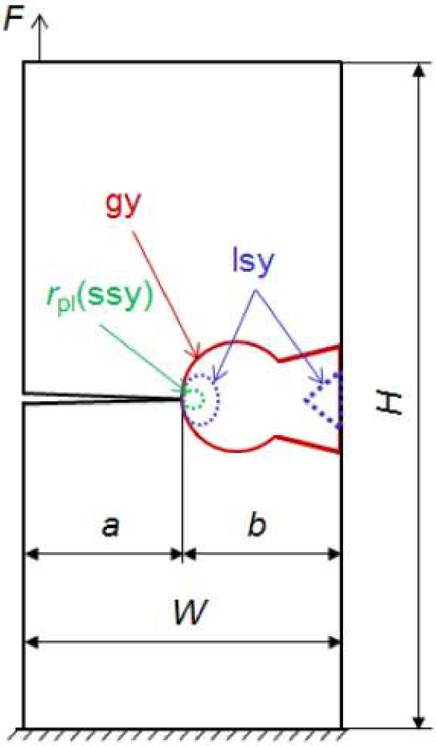

Figure 1 The loaded body with a crack of length a. The yielding conditions are distinguished by colours: green denotes small-scale yielding, blue large-scale yielding and red general scale yielding.

2.1. *Crack driving force in LEFM*

The two following approaches are strictly restricted to LEFM and the ssy regime resp. – elastic energy release rate G and stress intensity factor K.

2.1.1. The elastic energy release rate

The elastic energy release rate concept is useful for hard metals, composites with metal and ceramic matrix, hard strength metals and for materials where the plastic zone is negligible (LEFM or ssy regime) and is not changing during the crack growth. Then the crack growth resistance is equal twice specific surface energy γ_s and a specific plastic work for building the fracture surface γ_{pl} [1]:

$$R_{frs} = 2\gamma_{frs} = 2(\gamma_s + \gamma_{pl}). \tag{3}$$

The elastic energy release rate is expressed for a small interior crack of length $2a$. Loaded by the remote stresses σ_{appl} in an infinite plate as [1]:

$$G = \frac{\pi \sigma_{appl}^2 a}{E^{'}}, \tag{4}$$

where $E^{'}$ for plane strain is expressed as $E^{'} = E/(1-\nu)$ and for plane stress $E^{'} = E$ with E as the Young modulus and ν as the Poisson number. The critical length for a given applied stress can be calculated as:

$$a_{crit} = \frac{2\gamma_{frs} E^{'}}{\pi \sigma_{appl}^2}. \tag{5}$$

Thus, the critical applied stress for a given crack length is:

$$\sigma_{appl,crit} = \sqrt{\frac{2\gamma_{frs} E^{'}}{\pi a}}. \tag{6}$$

2.1.2. The stress intensity factor

The stress intensity factor K is dependent generally on the applied load (stress), the geometry of the body and the crack length a [1]:

$$K = \sigma_{appl} \sqrt{\pi a}\, f_k \left(\frac{a}{W}, \frac{H}{W}\right), \tag{7}$$

where $\sigma_{appl} = F/(BW)$ is the nominal stress, B is the thickness, W is the width and H is the height of the sample, see Figure 1. The crack starts to grow analogously to equation (1) if the stress intensity factor (applied stress tip field) K is equal or higher than the critical stress intensity factor K_c. Assuming plane strain state condition, the thickness of sample must be substantial larger than the radius of the plastic zone and is according to [1] expressed as (see also equation (2)):

$$a, b, B \geq 2.5 \frac{K_c^2}{\sigma_{YS}^2}, \tag{8}$$

where σ_{YS} is the yield strength of the material.

The stress intensity factor range ΔK can be in many cases related to the crack extension per load cycle of a fatigue crack da/dN but again LEFM (ssy) must be applicable [2].

2.2. *Crack driving force in E-PFM*

2.2.1. The crack tip opening displacement (CTOD)

CTOD characterizes the intensity of the near-tip field also in cases where LEFM is not applicable [1]. The crack grows in analogy to equation (1) if CTOD δ is equal or higher than critical CTOD δ_c. The δ_c is taken at $\Delta a = 0.2mm$. To characterize the intensity of the near-tip field

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via CTOD the crack and the sample length as well as the sample thickness must be much larger than δ_c [1]:

$$a, b, B \geq \delta_c. \tag{9}$$

The cyclic crack tip opening displacement, $\Delta\delta_t = \delta_{t,max} - \delta_{t,min}$, can be expressed in form of the stress intensity factor range ΔK according to [2] as:

$$\Delta\delta_t = \propto \frac{(\Delta K)^2}{2E\sigma_y}, \tag{10}$$

where $\propto$ is a constant and is approximately equal to 0.5 for plane stress and 0.5 for plane strain, σ_y is the yield stress.

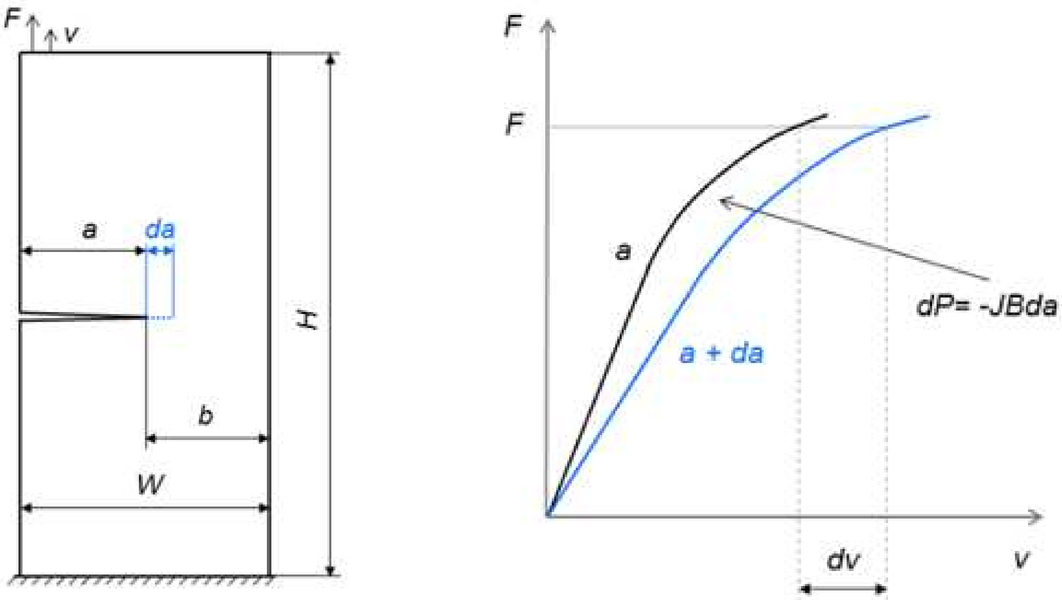
2.2.2. The J-integral

The J-integral measures the difference of the potential energy of two identical non-linear elastic bodies with different crack lengths, see Figure 2. Following [14], assumed is a homogeneous body with non-linear elastic material properties. Plane strain condition and no volume forces are imposed on the body. The crack lies in x-direction in the body. For those assumptions the J-integral is [1], [14]:

$$J = \int_\Gamma \left(\Phi dy - T_i \frac{\partial u_i}{\partial x} ds\right) = -\frac{1}{B}\frac{dP}{da}, \tag{11}$$

where Γ is the curve around the crack tip, ds is the element on the curve Γ, u_i, is the displacement vector, T_i is the traction vector, P is the potential energy of the body. The deformation energy in the body is defined as [1], [14]:

$$\Phi = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij}. \tag{12}$$



a) b)

Figure 2 Non-linear elastic body with the crack of two lengths (a). The J-integral is measure of a difference in the potential energy (b).

Similarly to the previous methods, crack grows occurs if $J \geq J_c$. For the J-concept to be valid J_c requires much less strict criteria have to be satisfied to the restrictions of the K or the CTOD concept, see equations (8, 9) and compare with equation (13), [1]:

$$B \geq l_{pz}, \tag{13}$$

where l_{pz} is the length of the process zone. l_{pz} is proportional to CTOD, $l_{pz} \approx (2 \div 3)\delta$. Thus this method is useful for a correct determination of the crack growth resistance J_c of a low strength material. The required sample size is in order of magnitude smaller than what is necessary for a correct determination of K_c.

Hutchinson, Rice and Rosengren showed that J characterizes the intensity of the crack tip field for elastic-plastic materials (so-called HRR field). Deformation plasticity and power-law hardening are assumed. Then the strain energy density near the crack tip follows the relationship $\Phi \sim J/r$ and the stress and strain components are given by [1]:

$$\sigma_{ij} = \sigma_0 \left(\frac{EJ}{\alpha_{RO}\sigma_0^2 I_N r}\right)^{1/(N+1)} \tilde{\sigma_{ij}}(N,\theta), \tag{14}$$

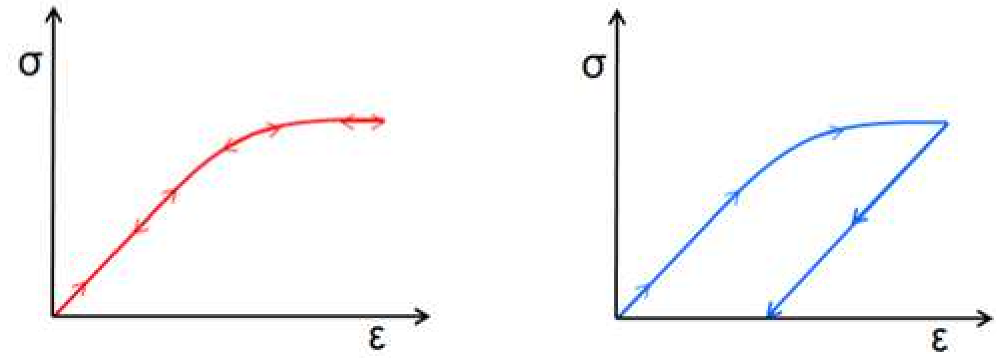
$$\varepsilon_{ij} = \frac{\alpha_{RO}\sigma_0}{E}\left(\frac{EJ}{\alpha_{RO}\sigma_0^2 I_N r}\right)^{N/(N+1)} \tilde{\varepsilon_{ij}}(N,\theta), \qquad (15)$$

where the stresses and strains are dependent on the polar coordinates (r,θ) with respect to the crack tip, α_{RO} is a dimensionless constant, σ_0 is a reference stress and is equal to the yield strength for a small α_{RO}, ε_0 is a reference strain ($\varepsilon_0 = \sigma_0/E$), N is a hardening parameter with value of 1 for a linear elastic material description and ∞ for an ideally plastic material description. The parameter I_N and the functions $\tilde{\sigma_{ij}}$, $\tilde{\varepsilon_{ij}}$ are tabulated as a function of N.

The conventional J-integral is not appropriate for non-proportional cyclic loading and does not characterize the real crack driving force for a crack in an elastic-plastic material [4]. The experimental cyclic J-integral ΔJ^{exp} was proposed as a parameter characterizing the crack growth rate da/dN of fatigue cracks for cases where ΔK is not applicable anymore.

2.2.3. The configurational force concept

The configurational force concept is based on a thermodynamic framework and Eshelby´s energy momentum tensor. The concept is able to account for an incremental theory of plasticity. Another concept, the previously mentioned J-integral is developed based on deformation theory of plasticity. It has been shown in [2] [15] [16] that a calculated crack driving force based on the deformation theory leads to incorrect results in cyclic loading. The principle difference between the two theories of plasticity is shown in Figure. 3.



a) b)

Figure 3 The stress-strain behaviour of the material described by deformation theory of plasticity is (a); the behaviour using incremental theory of plasticity (b).

The plastic strain in the deformation theory of plasticity is described as a function of equivalent strain $\varepsilon_p \approx \sigma_{eq}$, which is in effect a description for a non-linear elastic material, see Figure 3a and [1]. The incremental theory of plasticity describes an increment of plastic strain as a function of the equivalent stress: $d\varepsilon_p \approx \sigma_{eq}$, see Figure 3b and [1]. The two descriptions of the plastic strain results in different energy considerations as can be seen in the area below the stress-strain curve, see Figure 4 and [15]. The whole strain energy density Φ below the stress-strain curve can be divided into an elastic part Φ_{el} and a plastic part Φ_{pl}. Then the configurational body force vector can be written for the deformation theory of plasticity (non-linear elastic material description) as:

$$f^{nlel} = -\nabla(\Phi I - F^T S) \qquad (16)$$

and for the incremental theory of plasticity as:

$$f^{ep} = -\nabla(\Phi_{el} I - F^T S), \qquad (17)$$

where I is the unit tensor, F^T is the transposed deformation gradient tensor, S is the first Piola-Kirchhoff stress tensor.

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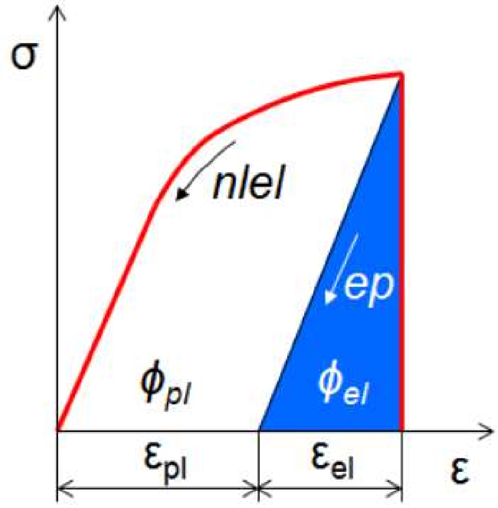

Figure 4 The energy consideration below the stress-strain curve. The whole area below the red curve represents the energy consideration according to the deformation theory of plasticity. The blue part represents the energy consideration of the incremental theory of plasticity. The plastic part of the strain energy density (white) is consumed during plastic deformation and only the elastic part is available for driving the crack.

The thermodynamic crack driving force for elastic-plastic materials along an arbitrary integration contour Γ_Γ (here shown for a crack driving force which originates from the crack tip contour $\Gamma_\Gamma \to 0 \Rightarrow J_{tip}^{ep}$ is expressed as [15]:

$$J_{tip_i}^{ep} = e_i \int_{\Gamma_\Gamma} (\Phi_{el} I - F^T S)\, \mathrm{m}\mathrm{dl}, \qquad (18)$$

where e_i is the unit vector related to the chosen direction of the local coordinate system of the crack, see Figure5; m is the unit normal vector on the integration contour Γ_Γ and dl is the infinitesimal small length on the circumference Γ_Γ.

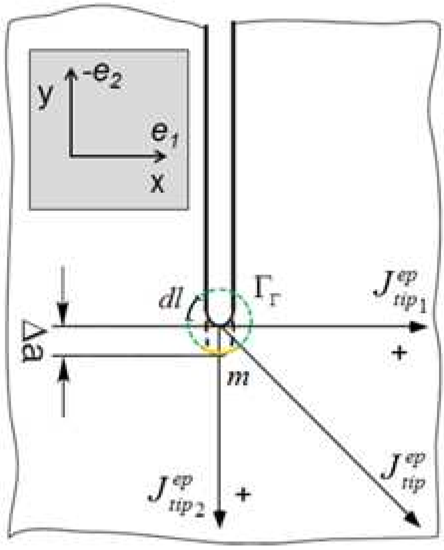

Figure 5 Sketch of the crack driving force vector which originates from the crack tip. The unit vectors are chosen in order to plot the crack driving force as function of the crack modes. Δa represents a virtual crack growth extension only when the crack driving force related to the crack Mode I $J_{tip_2}^{ep}$ supports crack growth.

3. Evaluation of the crack driving force approaches

The evaluated fracture mechanics (crack driving force) approaches are summarised in Table 1. The energy release rate and the stress intensity factor are restricted to LEFM or SSY, while other approaches can be used in E-PFM. The configurational force concept is not restricted for monotonic loading and can be used for arbitrary (quasi-static) loading and yielding conditions (denotes as cyclic plastic loading in the Table 1).

Table 1 Regime of the validity of various fracture mechanics approaches

	Loading Parameter for the Crack	**Regime of Validity**
Linear elasticity	**Elastic energy release rate** *G* (J/m²)	• linear elastic fracture mechanics (LEFM) • small-scale yielding
	Stress intensity factor *K* (Nm$^{-3/2}$)	• LEFM • small-scale yielding
General usage	**Crack tip opening displacement** (CTOD) *δ* (mm)	• small-scale yielding • large-scale yielding • general yielding
	J-integral *J* (J/m²) *No cyclic plastic loading*	• LEFM • small-scale yielding • large-scale yielding • general yielding
	Configurational force concept J^{ep} (J/m²) *Cyclic plastic loading*	• LEFM • small-scale yielding • large-scale yielding • general yielding

Conclusion

Crack driving force concepts are described and evaluated in regard to LEFM and E-PFM in the paper. The regime of their validity are summarised in the table. The configurational force concept is able to handle with arbitrary (quasi-static) loading and yielding conditions. In the introduction is discussed limitation of the concept in the practical applications.

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