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NON-DESTRUCTIVE INVESTIGATION OF FLEXURAL PROPERTIES OF CIRCULAR METAL AND PLASTIC HOOPS - - BY MEANS OF DOUBLE FLYWHEELS METHODS

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Abstract: We have suggested a new non-destructive method for measuring of tensile modulus of circular and hoop samples – by means of vertically oriented flywheel set. We had performed a theoretical analysis of its operation and we carried out the testing measurements of simple metal and plastic samples. The results being achieved are compared with the analogous values obtained using the horizontal device.

1 Introduction

The tensile modulus E (also called *elastic modulus* or *Young modulus*) is in here the centre of interest. This quantity belongs to the most important material constants; we can say it is a measure of the stiffness of matter. It determines the relation between stress σ along the axis, and strain ε at axial loading, in the form $\sigma = E.\varepsilon$, which is valid in the range of Hooke's law.

There exist several possibilities how to measure this quantity. The most commonly used way is the static method **-** by stretching /shortening the sample under its force stress. In the case of thin non-linear specimens, such as bent wires, sticks, rods, columns, fibres, and the like (with arc, circular etc. shape) however, its use is problematic; a permanent shape deformation of the material may occur. In such cases, it is advantageous to use some of the dynamic non-destructive methods that are based on the investigation of the intrinsic vibrations of the substances.

One such classical device is a horizontal flywheel set (also known as *Searle's pendulum*). Basically, here are sample oscillations at the three-point bend, which are still damped by flywheels. However, it should be noted that this method is only applicable to partially circular (it means arcuate) samples. For full circular and hoop patterns, the situation is more complicated.

And just creating a device for full circular and hoop samples was the goal of our work. For this purpose, we designed a vertically oriented flywheel assembly.

2 Horizontal coupled flywheel set as an interface to vertical assembly

Since the horizontal device represents a starting step on the path to the vertical set, we will describe it in more details, and the corresponding references and comparison will be related to it, according to scheme

partially circular samples——→ *fully circular samples*

and

horizontal set ——→ *vertical set.*

Horizontal structure consists of three main parts (Figure 1). There are two horizontal flywheels (mostly cylindrical) 2, fixed to two hinge yarns 1. They are connected by the measured arc-shaped sample 3; this one basically represents the element of "coupling".

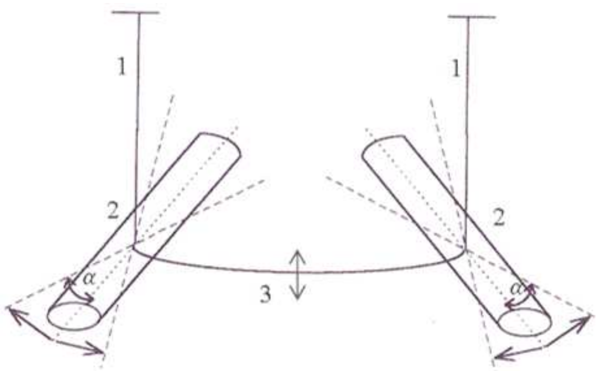

Figure 1 Horizontal flywheel set.
1 – hanging threads, 2 – cylinder flywheels, 3– measured wire (arrows indicate the direction of the oscillations)

Symmetrical deflection of the flywheels in the horizontal direction by the angle α performs the bending oscillating movement of the sample, that is reversely transmitted to the oscillating rotary motion of flywheels - and vice versa. Both parts of a pendulum – i.e. flywheels and sample - oscillate synchronously, with the same frequency and phase. Tensile modulus E can be calculated from a relationship [1], [2] as

$$E = \frac{8\pi l J}{r^4 T^2}, \tag{1}$$

wherein l is the length of the sample with the radius r. T means the oscillation period of the system, and J is the moment of inertia of the flywheel with respect to the perpendicular axis passing through the centre.

We present a detailed analysis of the measurement procedure using this device - including experimental measurements - in the article [3]. Here (in current paper) we report a comparison of results with vertical set (in Section 4.4 *Results of measurements*).

3 Our work

3.1 Aim of our work

We have suggested new equipment for examination of flexure properties of full circular samples by slowed oscillations – a vertically coupled gravitation set (also known as *coupled reverse pendulums*). Unlike the previous device, the slowing flywheels are located in the vertical direction; the purpose is also to use gravitational forces. A direction the measured sample can be arbitrary – horizontal or vertical; we used a horizontal way. However, the theoretical analysis is more complicated – we need to consider – except for gravity – also the contribution of next factors {see Section 3.3).

3.2 Measuring equipment

A sketch of equipment being used is shown in figure 2. Both reverse pendulums were hung so that they were vibrating in a common vertical plane. When using a classical spring connection for demonstration of composition of parallel oscillations, thus we can determine the spring's stiffness, too.

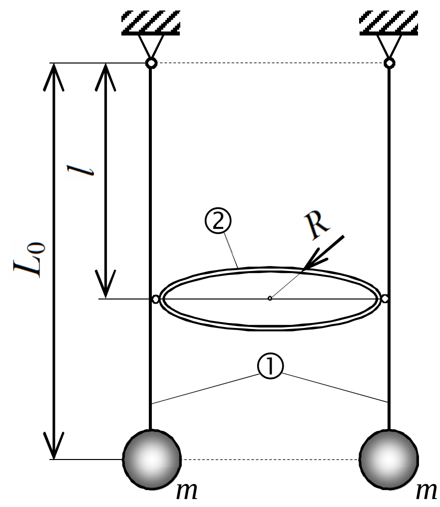

Fig. 2 Measuring equipment scheme.
(1 - pendulums, 2 – circular hoop connecting pendulums)

In our experiment an elastic wire shaped like a horizontal hoop was used as a connection. Deviation of the pendulums in their common plane gave rise to bending vibrations of the wire, while the same phenomenon as in the case of spring connection (i.e. energy transfer from one pendulum to another) could be observed.

Tensile modulus of the wire can be determined similarly as the spring stiffness can be specified. Corresponding basic circular frequencies ω_1 and ω_2 *necessary* for calculation can be determined by experiments from Fig. 3a and 3b examining concordant and/or discordant oscillations of the pendulums.

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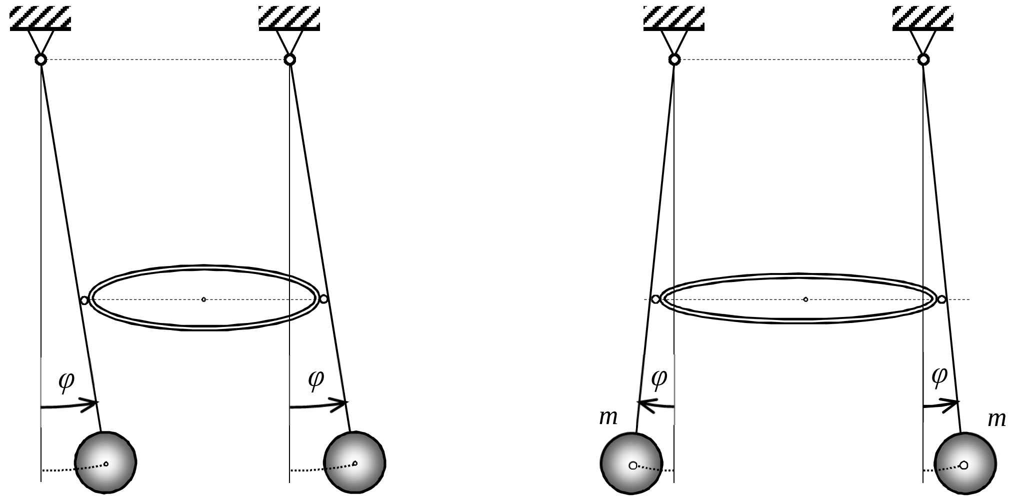


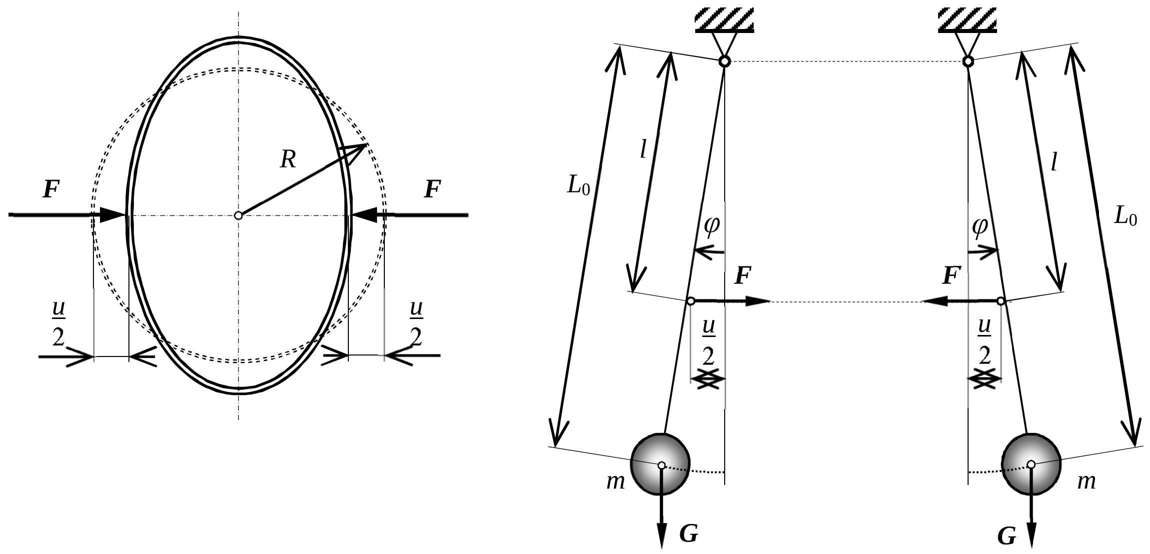
a) b)

Fig. 3 Vibrational modes of coupled pendulums
a) 1st mode - concordant vibrations, b) 2nd - discordant vibrations

3.3 Dynamic Analysis

In theoretical analysis - compared to a horizontal set - we need to incorporate into our calculations three significant realities:
- the pendulum forces are a mixture of elastic and gravitational activities
- the flywheels do not rotate around their central axes but around the pendulum ones
- the samples are full circular in shape and they are sufficiently thin (for shape deformation)

It is necessary to specify the range of the wire circular deformation caused by the force ***F*** (Figure 4).



a) b)

Fig. 4. Deformation diagram at transfer of force F: a) to the circular wire, b) to the pendulum.

(***G*** is weight of pendulum, *u* is a wire deformation; L_0 is distance of the pendulum centre from the rotation axis, *φ* is angle of the pendulum deviation and *l* is distance of the wire connection from the pendulum point).

To do so we used strain energy *A*, the quantity of which is given by bending effects in particular. Regarding perpendicular axes symmetry, the calculation was done only for a quadrant (Figure 5).

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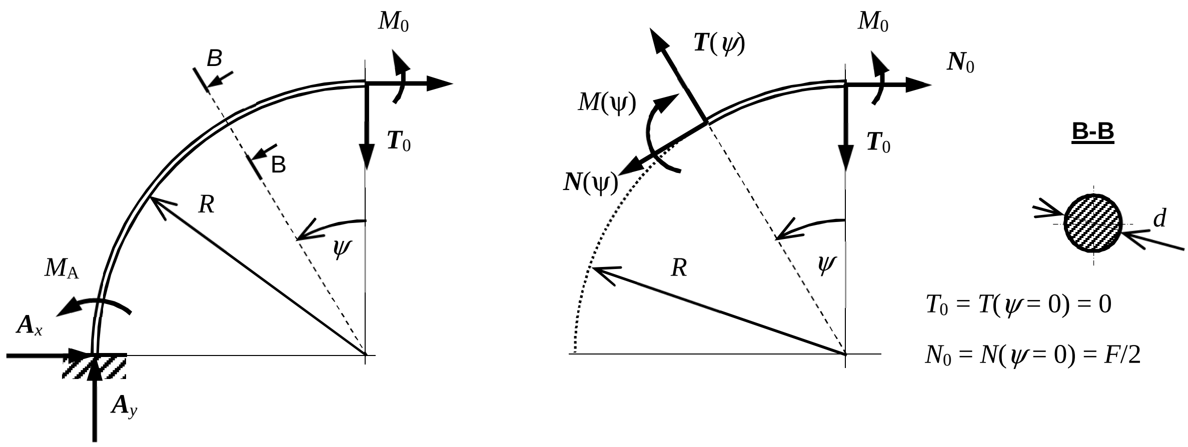

Fig. 5. The analysis of internal forces during deformation of circular wire

Strain energy A of the quadrant is as follows:

$$A = \frac{1}{2EJ_z}\int_0^{\frac{\pi}{2}} M^2(\psi) R \mathrm{d}\psi \tag{2}$$

where

$$M(\psi) = M_0 - \frac{F}{2} R(1-\cos\psi) \tag{3}$$

and E is tensile modulus, J_z is area moment of inertia about wire neutral axis, $M(\psi)$ is bending moment, R is arc radius and ψ represents the angle of turning of the arc. The values of T and N correspond to tangential and normal component of the force F.

Before the calculations it is necessary to determine the value of bending moment M_0, which corresponds to zero rotation at the point $\psi = 0$, i.e.

$$\frac{\partial A}{\partial M_0} = \frac{1}{EJ_z}\int_0^{\frac{\pi}{2}} \left[M_0 - \frac{F}{2} R(1-\cos\psi) \right] R \mathrm{d}\psi = 0 \tag{4}$$

$$M_0 = \frac{F}{2} R \frac{(\pi - 2)}{\pi} \tag{5}$$

The value of displacement u_1 at the point $\psi = 0$ can be determined from the following condition:

$$u_1 = \frac{\partial A}{\partial N_0} = \frac{\partial}{\partial N_0}\left[\frac{1}{2EJ_z}\int_0^{\frac{\pi}{2}} (M_0 - N_0 R(1-\cos\psi))^2 R \mathrm{d}\psi \right] \tag{6}$$

Total displacement u is the given by the equation:

$$u = 2u_1 = \frac{FR^3}{4EJ_z}\left(\frac{\pi^2 - 8}{\pi} \right). \tag{7}$$

Tensile modulus can be determined also from frequencies ω_1 and ω_2 of the connected pendulums. If the interaction between connecting circle element and pendulums is replaced by its force effect, then moment M applied on the pendulum (Figure 3b) can be determined as

$$M = mgL_0 \sin\varphi + Fl\,, \tag{8}$$

where m is pendulum weight and g is gravity acceleration.

Supposing that the pendulums are oscillating in the field of small oscillations ($\varphi < 5°$), $\sin\varphi \cong \varphi$ and $u = 2l\varphi$. Thus, using expressions (7) and (8) we can obtain a new relation:

$$M = mgL_0\varphi + \frac{8\pi l^2 EJ_z}{R^3(\pi^2 - 8)}\varphi \tag{9}$$

Setting this relation into motion equation of the pendulum we can calculate circular frequency for discordant oscillations of the connected pendulums:

$$\omega_2^2 = \frac{1}{I}\left[mgL_0 + \frac{8\pi l^2 EJ_z}{R^3(\pi^2 - 8)} \right] \tag{10}$$

where $I = mL_0^2$ is the pendulum inertia moment. A similar relation applies for circular frequency of concordant vibrations of two pendulums:

$$\omega_1^2 = \frac{mgL_0}{I} \tag{11}$$

Having treated the relations (10), (11) and using vibration periods $T_1 = 2\pi/\omega_1$, $T_2 = 2\pi/\omega_2$ and the well-known relation for area moment of inertia (with respect to the axis lying in the bending plane)

$$J_z = \frac{\pi d^4}{64} \tag{12}$$

where d is a wire diameter, we can obtain final relation for calculating tensile modulus of wire in the form of:

$$E = \frac{8(\pi^2 - 8) mgL_0 R^3}{\pi^2 l^2 d^4}\left[\frac{T_1^2}{T_2^2} - 1 \right] \tag{13}$$

3.4 Results of measurements

The measurements were carried out by means of connected pendulums as shown in figure 2. We have investigated the elastic properties of several materials – metallic and plastic - all with the same geometric parameters (length *l* and diameter *d*). These parameters are summarized in table 1.

Table 1 Geometrical parameters used in our experiments

L_0 [m]	m [kg]	d [mm]	R [m]	l [m]
0.84	0.87	1.5	0.16	0.25

Also the period of concordant vibrations was the same for all materials - $T_1 = 1.040$ s.

So, the only varying parameter had been the period of discordant vibrations T_2. The corresponding results for periods of discordant vibrations are summarized in table 2, together with the relevant values of tensile modules E calculated from the formula (13).

Table 2 Quantities measured for the determination of elastic modulus.

Sample	T_2 [s]	E [GPa]	E_{tab} [GPa]
Steel	0.610	203.3	200 - 210
Aluminium	0.808	69.9	67 - 70
Copper	0.707	124.1	110 -120
Brass	0.759	101.7	90 - 100
Polyamide (nylon)	1.034	1.3	0.9 – 1.4
Polystyrene	1.031	1.8	1.5 - 1.8

As we can see the results being obtained are in good agreement with material-table values (last column); the differences from mean table values represent no more as several per cents.

We also measured - for comparison purposes - the specimens using a classic horizontal assembly. The results obtained were similar to the measurements described, their deviation was a maximum of 10 % compared to the vertical set.

3.5 Statistical evaluation

Another evaluation can be presented in terms of statistical view, based on the accuracy of the single quantities being measured. Therefore, we made a statistical evaluation of the uncertainties, according to the degree of accuracy of the measurements of the individual variables.

It is a rather complicated statistical task. Here we must take into account that the tensile modulus - as can be seen from equation (13) – is a function of four quantities x_i being measured directly, namely $E = f(R,l,T_1,T_2)$; the values of m, L_0 and d were entered directly by the manufacturer and we considered them as constants. In this case – in accordance with theory of measurements - the uncertainty is given by a root, containing partial derivatives with respect to all of the relevant variables and uncertainties of these variables:

$$u_E = \pm\sqrt{\sum_{i=1}^{n}\left(\frac{\partial E}{\partial x_i}u_{x_i}\right)^2} = \pm\sqrt{\left(\frac{\partial E}{\partial R}u_R\right)^2 + \left(\frac{\partial E}{\partial l}u_l\right)^2 + \left(\frac{\partial E}{\partial T_1}u_{T_1}\right)^2 + \left(\frac{\partial E}{\partial T_2}u_{T_2}\right)^2} \tag{14}$$

.

The relevant partial derivatives of (14) are

$$\frac{\partial E}{\partial R} = \frac{24(\pi^2-8)mgL_0R^2}{\pi^2 l^2 d^4}\left[\frac{T_1^2}{T_2^2}-1\right]$$

$$\frac{\partial E}{\partial l} = \frac{-16(\pi^2-8)mgL_0R^3}{\pi^2 l^3 d^4}\left[\frac{T_1^2}{T_2^2}-1\right]$$

$$\frac{\partial E}{\partial T_1} = \frac{16(\pi^2-8)mgL_0R^3}{\pi^2 l^2 d^4}\frac{T_1}{T_2^2}$$

$$\frac{\partial E}{\partial T_2} = \frac{-16(\pi^2-8)mgL_0R^3}{\pi^2 l^2 d^4}\frac{T_1^2}{T_2^3}.$$

Here we have set up the precision of measuring instruments for applying the uncertainties of them (e.g. u_R, u_l, u_{T1} and u_{T2}) as the size of the smallest pieces on their scales. So:

$u_R = 1$ mm (ruler)
$u_l = 1$ mm (ruler)
u_{T1} , $u_{T2} = 0{,}01$ s (stopwatch)

After fitting all the variables being relevant we shall get a final value for the resulting uncertainty $U_E \div 8$ %. Thus, the final result can be written – for steel, for example – as

$E = 203.3$ GPa ± 8 %.

4 Conclusion

The described equipment is simple and illustrative, completing the range of pendulum-based methods for the measurements of elasticity constants. Regarding below 10 % accuracy it ranges to the (relatively) accurate methods. It does non-require intricate measuring equipment and works without destruction, practically. Even extremely thin samples can be measured without a risk of damage or permanent deformation. The activity of pendulums is stable, the system phases do not “tune out” or dump even after several tens or hundreds of oscillations. This method can be successfully used as a demonstration specimen in a university textbook (chapter “Vibrating Movements” or “Solids Physics”), or a task for laboratory exercises.

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